Abstract. The hypothesis is advanced that after collision of a Mars-sized impact with the Earth, collisions between debris particles themselves are able to place enough material into Earth orbit, to form the Moon. Collision probability estimates show that the collision frequency is high enough to place about one lunar mass into Earth orbit, if the average semimajor axis is of order of the Earth’s Roche limit of 18 500 km.

The large impactor hypothesis requires that a Mars-sized body impacts the Earth under special circumstances, and an expanding gaseous fireball resulting from the collision, injects enough material into Earth orbit beyond the Roche limit at height $\approx 18 \, 500 \, \text{km}$ to form the Moon (e.g. Wood, 1986; Boss and Peale, 1986). While the special near-grazing collision of a Mars-sized object has an acknowledged low probability, the expansion of an assumed hot gas-ball up to the Roche limit, forming the Moon via viscous coupling, gravitational instability, partial condensation, etc., seems at best questionable to me. Besides, cosmochemical arguments concerning fractionation of rare-earth elements in the Moon indicate that vapor fractionation was limited to temperatures less than 1100 K, and probably even lower (Wood, 1986). Thus, no selective loss of mass from proto-lunar matter did occur at temperatures above 1100 K, and does not favour lunar formation by cooling from a hot gas ball up to temperatures of order 270 K.

Therefore I return to Opik’s (1971) original suggestion that the Moon formed due to near-grazing collisions between Earth and massive planetesimals. To estimate the probability of near-grazing collision, we use the distribution function $\sin 2i$ for the impact angle $i$ (Shoemaker, 1962) to find the probability that the impact angle is contained between $\pi/2$ and $i$:

$$p_i = \frac{\int_{-\pi/2}^{\pi/2} \sin 2i \, di}{\int_{-\pi/2}^{\pi/2} \sin 2i \, di} = \frac{1 + \cos 2i}{2}. \quad (1)$$

We take somewhat deliberately $i = 60^\circ$ as the lower limit of near-grazing impacts, and find in this case $p_i = 0.25$. Thus, about 1/4 Earth masses or $1.5 \times 10^{27}$ g are expected to collide with the growing Earth at impact angles $> \pi/3$. If $i > 80^\circ$, there results $p_i = 0.03$, and $1.8 \times 10^{26}$ g impact the Earth at impact angles $> 80^\circ$. Impact velocities on Earth are above the escape velocity of 11.2 km s$^{-1}$, and probably of order 15 km s$^{-1}$ (Horedt, 1985). If the ejection velocity of collisional debris (composed of matter from the Earth and the planetesimal) is larger than the circular
satellite velocity \((7.9 \text{ km s}^{-1})\) at the surface of the Earth, it leaves the neighbourhood of the Earth on ballistic trajectories. It is commonly assumed that collisional debris is either reaccreted by the Earth (if its velocity is \(<11.2 \text{ km s}^{-1}\) ), or leaves the Earth’s surroundings on a heliocentric trajectory. This view ignores the possibility of collisions among the debris after a near-grazing collision with the Earth, resulting in orbital changes of debris particles that lead eventually to satellite-like orbits.

It should be emphasized that the precise outcomes of near-grazing collisions are difficult to model, so I make only rough worst and best case estimates, showing that injection of one lunar mass into Earth orbit by mutual collisions of debris particles is possible. The number of collisions that a debris particle experiences during the time interval \(t\) in a medium with the number density of particles \(n\) is given by (e.g. Feynman et al., 1963)

\[
N_c = 4\pi r^2 nut = 4\pi r^2 NUt/V,
\]

where \(r\) is the average radius of debris particles, \(U\) the relative velocity among the particles and \(N\) the number of particles in volume \(V\). In the worst case estimate we can assume for the collision volume \(V\) as an upper limit half of the Earth’s gravitational sphere of action, taking into account that debris particles will be ejected by near-grazing impacts mainly in the impact direction (Shoemaker, 1962, p. 343; 1983, p. 473). The radius of the gravitational sphere of action is (e.g. Horedt, 1985)

\[
R_G = a_p (m_p/2M)^{1/3} = 1.72 \times 10^6 \text{ km},
\]

\(a_p\) being the Earth-Sun distance and \(m_p\), \(M\) the mass of the Earth and of the Sun, respectively. The semimajor axis \(a_s\) of debris particles is taken in this case equal to \(a_s = R_G/2 = 8.6 \times 10^5 \text{ km}\), allowing for particle orbits with planetocentric eccentricity \(e_s \approx 1\). As a lower limit for \(a_s\) we take the Earth’s Roche limit (e.g. Jeans, 1961)

\[
a_s = 2.4554 \left( \rho_p/\rho_m \right)^{1/3} r_p = 2.9 r_p = 18 \text{ 500 km},
\]

and the corresponding radius of the hemispherical collision volume equals \(2a_s = 37 \text{ 000 km}\). \(\rho_p\) and \(\rho_m\) denotes the present-day mean density of the Earth and of the Moon, and \(r_p\) the Earth radius. The collision volume is given by

\[
V = (1/2) \times 4\pi (2a_s)^3/3 = 16\pi a_s^3/3.
\]

The time interval available for collisions can be taken equal to the revolution period round the Earth of a debris particle having semimajor axis \(a_s\)

\[
t = 2\pi a_s^{3/2}/(Gm_p)^{1/2},
\]

where \(G\) is the gravitational constant, and if \(a_s = 860 \text{ 000 km and 37 \text{ 000 km}}\), we obtain \(t = 91.6\) and 0.300 days. The average relative velocity between debris particles is subject to large uncertainty and we take \(U = 0.1\) and 1 \text{ km s}^{-1}; for comparison, the escape velocity at the present-day orbit of the Moon is 1.44 \text{ km s}^{-1}. The largest uncertainty (besides the value of the collision volume \(V\)) involves the number \(n\) of debris particles, and the resulting average radius \(r\) of debris particles caused by a Mars-sized impactor of mass \(m = 6.42 \times 10^{26} \text{ g}\) and density \(\rho_s = 3.34 \text{ g cm}^{-3}\), equal