Options Traders Exhibit Subadditive Decision Weights

CRAIG R. FOX
Fuqua School of Business, Duke University, Box 90120, Durham, NC 27708-0120

BRETT A. ROGERS
Department of Psychology, Stanford University, Jordan Hall, Building 420, Stanford, CA 94305-2130

AMOS TVERSKY
Department of Psychology, Stanford University, Jordan Hall, Building 420, Stanford, CA 94305-2130

Abstract

Professional options traders priced risky prospects as well as uncertain prospects whose outcomes depended on future values of various stocks. The prices of the risky prospects coincided with their expected value, but the prices of the uncertain prospects violated expected utility theory. An event had greater impact on prices when it turned an impossibility into a possibility or a possibility into a certainty than when it merely made a possibility more or less likely, as predicted by prospect theory. This phenomenon is attributed to the subadditivity of judged probabilities.

Key words: risk, uncertainty, decision weights, subadditivity

In recent years, several authors have attempted to accommodate violations of expected utility theory by replacing additive probabilities with nonadditive decision weights (e.g., Kahneman and Tversky, 1979; Quiggin, 1982; Yaari, 1987; Gilboa, 1987; Schmeidler, 1989; Wakker, 1989; Luce and Fishburn, 1991; Tversky and Kahneman, 1992). Empirical evidence suggests that decision weights follow a distinctive pattern: an event has greater impact when it turns impossibility into possibility or possibility into certainty than when it merely makes a possibility more or less likely. This principle, called bounded subadditivity (Tversky and Wakker, 1995), can explain the certainty effect, the possibility effect, and other violations of expected utility theory (see Wu and Gonzalez, 1996). Tversky and Fox (1995) tested this principle in a series of studies involving risky prospects as well as uncertain prospects whose outcomes depended on upcoming sporting events, future temperatures, and future values of the Dow-Jones index. Bounded subadditivity was observed for both risk and uncertainty, and this phenomenon was more pronounced for uncertainty than for risk.

The participants in these studies were Stanford students recruited specifically for their sports acumen; some were actually paid on the basis of their responses. The question arises whether expert decision makers who evaluate uncertain prospects as part of their daily professional activity also exhibit bounded subadditivity in their domain of expertise. To answer this question, we recruited professional options traders and support staff on the
floors of the Pacific Stock Exchange and Chicago Board Options Exchange, and asked them to price both risky prospects and uncertain prospects based on future values of various stocks. Options traders can be considered experts in decision making under uncertainty. They are schooled in the calculus of chance; they have a great deal of experience assessing uncertainty; they are trained to identify and exploit arbitrage opportunities; and they are selected for their ability to do so. Consequently, options traders—perhaps more than other groups—could be expected to avoid the biases commonly observed in studies of decision under uncertainty.

1. Theory

We first introduce the theoretical framework of cumulative prospect theory that underlies the analysis presented in this article (Tversky and Kahneman, 1992; Wakker and Tversky, 1993). Because we consider here only prospects with nonnegative outcomes, this representation is essentially equivalent to the rank-dependent utility model. Let $S$ be a set whose elements are interpreted as states of the world. Subsets of $S$ are called events. Thus, $S$ corresponds to the certain event, and $\emptyset$ is the null event. A weighting function $W$ (on $S$) is a mapping that assigns to each event in $S$ a number between 0 and 1 such that $W(\emptyset) = 0$, $W(S) = 1$, and $W(A) \geq W(B)$ if $A \supset B$. Such a function is also called a capacity, or a nonadditive probability.

Let $(x_1, A_1, \ldots, x_n, A_n)$ be a prospect that offers $x_i$ if event $A_i$ obtains. Assume that $(A_1, \ldots, A_n)$ is a partition of $S$ and that $0 \leq x_1 \leq \ldots \leq x_n$. The value of such a prospect is given by

$$
\sum_{i=1}^{n} v(x_i)\pi_i,
$$

where $\pi_n = W(A_n)$ and $\pi_i = W(A_i \cup \ldots \cup A_n) - W(A_{i+1} \cup \ldots \cup A_n)$,

$$
i = 1, \ldots, n - 1.
$$

Thus, the value of a nonnegative prospect is determined jointly by the value function for monetary gains $v$, and the weighting function $W$, defined on $S$. We assume that $v$ is strictly increasing and that $v(0) = 0$. Note that this form reduces to an expected utility model if $W$ is additive, that is, if $W(A \cup B) = W(A) + W(B)$, for $A \cap B = \emptyset$. Prospect theory, however, imposes the following weaker constraints.

1. **Lower subadditivity:** $W(A) \geq W(A \cup B) - W(B)$, provided $A$ and $B$ are disjoint and $W(A \cup B)$ is bounded away from one. This inequality captures the possibility effect: the impact of an event $A$ is greater when it is added to the null event than when it is added to some nonnull event $B$.

2. **Upper subadditivity:** $W(S) - W(S - A) \geq W(A \cup B) - W(B)$, provided $A$ and $B$ are disjoint and $W(B)$ is bounded away from zero. This inequality captures the certainty