SIMULATION OF EVOLUTIONARY MECHANISM
OF PLANETARY RINGS

HONGNAN ZHOU
Astronomy Department, Nanjing University Nanjing, China

(Received 19 June, 1989)

Abstract. We devised a numerical model of planetary ring in which the inelastic collision and the
gravitation between ring particles are considered. We adopt Hill's equations and the differential
algorithm of circular mesh for computation of the particle orbits. The evolutionary processes are
presented for different coefficient of restitution $e$ and dynamical optical depth $\tau$. The results show that
the semi-major axis and eccentricity of the ring particles are changed with $e$ and $\tau$. We compute the
average energies transferred and loss in inelastic collisions for various values of the parameters. The
dynamical equilibrium properties are discussed in the different cases.

1. Introduction

Usually, the structure and dynamical evolution of planetary ring is investigated as
an important part of the solar system's evolution. The collision and gravitation
between particles are the powerful mechanism in the evolution of the ring. The
numerical experiment of interaction of particles are tested by many authors –
such as Brahic (1977), Lin and Bodenheimer (1981), Goldreich and Tremaine
Duncan, Quinn and Tremaine (1988), etc.

In this paper, a numerical model of planetary ring is presented in Section 2. We
derive a calculational scheme in Section 3. In this scheme, the Hill's equations
and middle point different method are adopted, the collision and effect of
restitution between particles are discussed, the gravitational interaction of parti-
cles are computed by annual mesh different algorithm. Finally, the evolitional
properties of ring are illustrated in Section 4.

2. Ring Model

2.1. N-BODY APPROACH

We present a ring model by a N-body approach. The main hypotheses are: (1)
The system consists of a collection of N particles orbiting in a plane around a
planet; (2) The ring model processes azimuthal symmetry; (3) The mass and radii
of each particle remains constant throughout the evolution; (4) The dynamical
evolution is basically determined by collision and gravitational interaction be-
tween ring particles.

2.2. Initial distribution

The initial Keplerian orbits of ring particles are chosen at random in the ring system. Let $R_{\text{out}}$, $R_{\text{inn}}$ are outer radii and inner radii of ring, RAN is number of random in range $[0, 1]$. Then,

\begin{align*}
\text{semimajor axis} & \quad a = \text{RAN} \cdot (R_{\text{out}} - R_{\text{inn}}) + R_{\text{inn}}, \\
\text{eccentricity} & \quad e = \text{RAN} \cdot (0.0003 - 0.0001) + 0.0001, \\
\text{true anomaly} & \quad \theta = \text{RAN} \cdot 2\pi,
\end{align*}

from which the initial coordinates and velocities of ring particles can be obtained.

3. Simulation Experiment Method

3.1. HILL'S EQUATIONS

We establish a planar rotation Cartesian coordinate system with origin at the reference position, the $X$ axis pointing outward, the $Y$ axis points in the direction of the orbital motion. We assume that the reference point moves on a circular orbit with semimajor axis $a_0$ and radial distance $r_0$ from the planet. Thus, the equations of motion of ring particles relative to the point of reference may be written by Hill’s equations (Hill, 1978; Wisdom and Tremaine, 1988) of the form

\begin{align*}
\dot{x}_i - 2\omega \dot{y}_i - 3\omega^2 x_i &= (F_{ij})_x, \\
\dot{y}_i + 2\omega \dot{x}_i &= (F_{ij})_y \quad (i = 1, 2, \ldots, N; j = 1, 2, \ldots, N; i \neq j),
\end{align*}

where $F_{ij}$ is the force per unit mass due to the other ring particles. $\omega = \sqrt{Gm_p/a_0^3}$ is the mean motion of the reference point. Equation (4) can be written as a system of four first-order differential equations by introducing the velocity coordinate $u_i, v_i$: i.e.,

\begin{align*}
\ddot{x}_i &= u_i = F_1, \\
\ddot{y}_i &= v_i = F_2, \\
u_i &= 2\omega v_i + 3\omega^2 x_i + (F_{ij})_x = F_3 \quad (i, j = 1, 2, \ldots, N; i \neq j), \\
v_i &= -2\omega u_i + (F_{ij})_y = F_4.
\end{align*}

If we keep only the second-order term, Equations (6) can be replaced by the central-point difference equations

\begin{align*}
&x_i = (x_i)_0 + (x_i)_0 \, dt + (F_1)_0 \, dt/2, \\
&y_i = (y_i)_0 + (y_i)_0 \, dt + (F_2)_0 \, dt/2, \\
&x_i = (x_i)_0 + (F_3)_0 \, dt/2, \\
&y_i = (y_i)_0 + (F_4)_0 \, dt/2 \quad (i = 1, 2, \ldots, N).
\end{align*}