A MODIFIED TURBULENT ENERGY MODEL FOR GEOPHYSICAL FLOWS: INFLUENCE OF THE GROUND PROXIMITY

B. GALPERIN
Geophysical Fluid Dynamics Program, Princeton University, Princeton, NJ 08542, U.S.A.

and

S. HASSID
Environmental Engineering and Water Resources,
Technion-Israel Institute of Technology, Haifa 32000, Israel

(Received in final form 4 July, 1985)

Abstract. A modified form of the turbulent energy model for geophysical flows is presented, in which the effect of the proximity of the ground on the pressure redistribution terms is accounted for. The new form of the model retains the simplicity and the capabilities of the original form, but at the same time differentiates between the vertical and lateral components of turbulent energy in boundary layers as well as between free and near-wall turbulence.

1. Introduction

In a previous publication (Hassid and Galperin, 1983a, HG below) a simple turbulent energy model for geophysical flows, closely related to the 2.5-level model of Mellor and Yamada (1982), has been described and its capability to depict reasonably well various micrometeorological and laboratory flows has been demonstrated (Hassid and Galperin, 1983b, 1984). This model, however, implies that in neutral flows the vertical and lateral components of turbulent energy are equal, whereas the experimental data (for example, Klebanoff, 1955) suggest that $u_2 \approx 2w^2$ in boundary layers and $v^2 \approx w^2$ in free turbulence (Gibson and Launder, 1978). A theoretical explanation of this difference lies in the influence of the proximity of the wall on the turbulent pressure-velocity correlations, known as redistribution terms. This influence is strong in the immediate vicinity of the wall, which results in a decrease of the vertical fluctuation intensity, in favor of the streamwise and lateral components, but gets weaker at higher values of $z$.

In this work, the original HG model is modified so that the influence of wall proximity is accounted for, using a simple technique suggested by Shir (1973) and Gibson and Launder (1978). The modified form of the model will be checked against available experimental data both in neutral and in stratified flows.

2. The Model

For the derivation of the full set of the equations of the turbulent energy model, see HG. To include ground effects, the pressure-velocity and pressure-potential temperature
correlation terms in the Reynolds stress and turbulent heat flux equations will be modeled as follows:

\[
\frac{\partial}{\partial x_i} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = - \frac{q}{3A_1} \left( \frac{u_i u_j - \frac{q^2}{3} \delta_{ij}}{n_k r_k} \right) + \\
+ \left[ C_i (C_i - C_i) f \left( \frac{1}{n_k r_k} \right) \right] \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) q^2 \\
+ \frac{A_i}{A_1} \left( \frac{1}{n_k r_k} \right) (u_k u_i n_k n_i \delta_{ij} - \frac{3}{2} u_i u_i n_k n_i - \frac{3}{2} u_k u_i n_k n_i),
\]

(1)

\[
\frac{\partial}{\partial x_i} \left( \frac{\partial \theta}{\partial x_j} + \frac{\partial \theta}{\partial x_i} \right) = - \frac{q}{3A_2} \left[ \frac{u_i u_j - A_2 f \left( \frac{1}{n_k r_k} \right) u_k \theta n_k} \right].
\]

(2)

The model contains three new constants, \( C_i \), \( A_i \), and \( A \), as well as a wall proximity function, \( f(\theta/n_k r_k) \), which satisfies the boundary conditions \( f(0) = 1, f(\infty) = 0 \). As is usually done in turbulent flow models, the value of \( A_i \), \( A_2 \) and \( C_i \) will be determined from simple, well documented turbulent flows.

Using Equations (1) and (2), as well as the equilibrium forms of the Reynolds stress and turbulent heat flux equations, a set of equations can be derived which describes the components of the Reynolds stress and turbulent heat flux in terms of quantities which can be calculated from the turbulent energy equation:

\[
\frac{\partial}{\partial x_i} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{q^2}{3} \left[ \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] = - \frac{2A_1 f}{q} \left[ \frac{2u_i u_j \partial U_i}{\partial x_j} - \frac{u_i u_j \partial U_i}{\partial x_j} + \beta g w \theta \right] + 3A_i f \frac{u_i u_j}{w} \left( \begin{array}{c} 1 \\ -2 \end{array} \right),
\]

(3a, b, c)

\[
- \left( \frac{\partial u}{\partial x_i} \right) = \frac{3A_1 f}{q} \left( \left[ \frac{u_i u_j}{w} - [C_i + (C_i - C_i) f q^2] \partial \Phi / \partial x_j - \beta g w \theta \right] \left[ \frac{u_i u_j}{w} - [C_i + (C_i - C_i) f q^2] \partial \Phi / \partial x_j - \beta g w \theta \right] \right) + \frac{A_1 f}{w} \left( \begin{array}{c} 0 \\ 0 \end{array} \right),
\]

(4a, b, c)

\[
- \left( \frac{\partial \theta}{\partial x_i} \right) = \frac{3A_2 f}{q} \left( \left[ \frac{u_i u_j}{w} \partial \Phi / \partial x_j + \frac{w \theta \partial U_i}{\partial x_j} \right] \left[ \frac{u_i u_j}{w} \partial \Phi / \partial x_j + \frac{w \theta \partial U_i}{\partial x_j} \right] + 3A_2 f \frac{w \theta}{w} \left( \begin{array}{c} 0 \\ 1 \end{array} \right).\right.
\]

(5a, b, c)

The remaining equations of the model remain unchanged. This includes the equilibrium formulation for \( \Phi^2 \).

From Equations (3), (4), and (5), algebraic equations can be derived for the eddy diffusivity \( K_T \) and the eddy viscosity \( K_M \):

\[
a_{11} K_M + a_{12} K_T = b_1,
\]

(6a, b)

\[
a_{21} K_M + a_{22} K_T = b_2
\]