Abstract. A numerical model based upon fundamental principles, a standard \((k, \epsilon)\) turbulence closure and a finite element integration technique, is applied to separated flows over hills. Predictions are compared to experimental data from a wind tunnel. Although few-equation turbulence closures have been shown to have obvious deficiencies with respect to comparable flows, the model predicts remarkably accurately, without coefficient adjustments of any kind. Even the turbulent intensity is predicted quite realistically.

1. Introduction

This study is motivated by aviation safety in flows over mountainous terrain. The critical response of an aeroplane may be divided into two parts: the first is associated with structural failure, either because the actual turbulence is too strong or the aircraft has been exposed to turbulence over too long time. The second type is associated with insufficient control by elevator, rudder, ailerons and throttle. Sudden wind changes can also be difficult to detect rapidly enough. Aviation safety focuses upon extreme conditions and the most hazardous locations within inhomogeneous flows. Waves and turbulence from upstream hills may affect the local flow over a given hill. Nevertheless, it appears fruitful in some cases to assume that effects from upstream hills may be second–order corrections to the local flow over a dominant hill. The most relevant scale is therefore associated with the local flow over a dominant hill. Rational estimation of aviation safety requires flow models for this.

The flow over a hill is a classical scientific subject. Such flows can take infinitely many forms (Grimshaw and Yi, 1991; ?, 1993; Lamb, 1994a, 1994b; Rottman et al., 1994). There may be spectacular mountain waves, which may also break (Castro and Snyder, 1993; Paisley and Castro, 1994). There are mean flow speedup, large mean shear and intense large–scale turbulence associated with separation and hydraulic jumps (Hunt and Snyder, 1980; Long, 1972; Lawrence, 1993). Most scientific understanding comes from two–dimensional stratified flows. However, three–dimensional flows over hills have also attracted significant attention. Some analytical analysis is still possible, and simulations have been comprehended. Experimental knowledge from laboratory measurements and field measurements from atmospheric and ocean environments are essential to increase our understanding of such flows (Jenkins et al., 1981; Mickle et al., 1988; Snyder et al., 1985;
Gong and Ibbetson, 1989; Finnigan et al., 1990; Coppin et al., 1994; Almeida et al., 1993). A state of the art review on the knowledge of flows over hills is given by Taylor et al. (1987); Hunt et al. (1991) and Baines (1995). Numerical simulations of turbulent separated flows over three-dimensional hills have not been possible until quite recently (Raithby et al., 1987; Yang, 1993; Dornback and Schumann, 1993; Xu et al., 1994).

The numerical model applied in this study is based upon the Reynolds equations and a standard \((k, \varepsilon)\) turbulence closure. The present study is restricted to strong flows with fully developed turbulence so that the density stratification is not important. The main result is that the model predicts neutrally stratified flows over hills remarkably accurately and robustly, without any coefficient adjustments.

2. Numerical Model

2.1. Reynolds Equations

The Reynolds equations for conservation of mean mass and momentum are given by

\[
\frac{\partial u_i}{\partial x_i} = 0
\]

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} - g \delta_{i3} - \frac{\partial \langle u_i' u_j' \rangle}{\partial x_j}.
\]  

The model is fully three-dimensional, and the flow is represented in terms of expected velocity components \(u_i(x_1, x_2, x_3; t)\), for \(i = 1, 2, 3\). The coordinate system is commonly aligned with the \(1\)-axis along the surface wind and the vertical direction along the \(3\)-axis. The meteorological tradition of denoting the vertical component \(x_3 = z\) is also used. In Equation (2) the gravity acceleration is given by \(-g \delta_{i3}\), where \(\delta_{ij}\) is the Kronecker delta. The turbulent fluctuations are denoted by \(u_i'\), and the covariance term \(\langle u_i' u_j' \rangle\) gives the Reynolds stress.

Standard boundary conditions are applied. The inflow condition is represented by a fully developed boundary layer over a rough surface. This condition is also applied locally as a wall boundary condition. At the side boundaries a weak form of zero flux conditions are used.

The numerical formulation is based upon a Galerkin finite element method. For the Reynolds equations a fractional step formulation is applied. An intermediate velocity is first estimated by neglecting the pressure gradient, the pressure field is then estimated from a Poisson equation, and used to correct the velocity field. The intermediate velocity and the turbulence equations are solved by an explicit two-step Taylor-Galerkin formulation. This integration procedure is known to be well suited for separated flows (Utnes and Ren, 1995; Utnes et al., 1995).