Comments on ‘AN OBSERVATIONAL STUDY OF HEAT FLUXES AND THEIR RELATIONSHIP WITH NET RADIATION’,
by D. CAMUFFO and A. BERNARDI

(Correspondence)

H. LETTAU
Department of Meteorology. University of Wisconsin at Madison, U.S.A.

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Recently, Camuffo and Bernardi (1982) published a study aimed at finding a general time-dependent formulation of the fluxes of heat in the balance equation at the soil/air interface,

\[ \Phi = G + H + LE, \]  

where \( \Phi = \) net radiation, \( G \) and \( H \) = fluxes of sensible heat into the ground and the air, respectively, and \( LE \) = flux of latent heat into the air. They introduce what they call two 'general equations':

\[ G(t) = a_1 \Phi(t) + a_2 \frac{d\Phi}{dt} + a_3, \]  

\[ LE(t) = b_1 \Phi(t) + b_2 \frac{d\Phi}{dt} + b_3, \]  

where \( t \) is the independent variable of time and the six constants \( (a, b) \) are empirical coefficients; a fourth equation follows from (1) through (3),

\[ H(t) = (1 - a_1 - b_1) \Phi(t) - (a_2 + b_2) \frac{d\Phi}{dt} - (a_3 + b_3). \]  

The authors state that further work is needed to calculate the values of the coefficients on the basis of meteorological and soil variables.

My comments concern the fact that Equations (2) and (4) are inconsistent with the physical laws of two-media diffusion of sensible heat in the case of heat being released at the interface (height \( z = 0 \)). Convection (at \( +z \) and \( t \)) and conduction (at \( -z \) and \( t \)) produce temperature profiles in the two media that must be continuous at \( z = 0 \), but can and must be discontinuous in their first-order derivatives \( (dT/dz) \). While Equation (3) is useful for defining the effective forcing \( F = \Phi - LE \), i.e., radiant energy minus that fraction used for generation of latent heat, Equations (2) and (4) must be replaced by rigorous consequences of 'First-Order Thermal Response' (Lettau, 1977),

\[ G(t) = r_G[T(t) + t_G \frac{dT}{dt}] + c_G, \]  

\[ H(t) = r_H[T(t) + t_H \frac{dT}{dt}] + c_H, \]  

where \( T \) is the temperature at the soil/air interface, the \( r \)-factors are 'respondances', the \( r^* \)-factors 'retention-time' or 'delay-time' values, and the constants \( c \) are to be determined.

by boundary conditions. This set of factors is not of an empirical nature insofar as it can be directly related to soil and meteorological variables.

In the special case of homogeneous soil, volumetric heat capacity $C$ as well as heat conductivity $\lambda$ are independent of both $z$ and $t$, and if surface heat flux $G(t)$ were to vary with a single harmonic of frequency $\omega$, Fourier's law combined with the law of heat continuity would yield from Equation (5):

$$r_G = [\lambda C \omega]^{1/2} \cos \alpha = \tan(\theta)\omega,$$

where $\alpha$ = phase constant by which the temperature cycle lags behind the $G(t)$ cycle; in homogeneous soil, $\tan \alpha = 1$, since $\alpha = 45^\circ$, or 3 hr for diurnal cycles. For example, when $\omega = 2\pi/86400$ s = $2\pi/24$ hr, using the data on Table 2.1 in Oke (1978) for sandy soil: $C = 1.28$ J cm$^{-3}$ deg$^{-1}$ and $\lambda = 0.3$ W m$^{-1}$ deg$^{-1}$, yields $r_G = 3.74$ W mm$^2$ deg$^{-1}$ and $r_{G*} = 3.82$ hr in Equations (7).

In the atmosphere, the corresponding two fundamental coefficients always depend on both $z$ and $t$; volumetric heat capacity $(c_p \rho)$ varies weakly but thermal diffusivity $(K)$ varies strongly, increasing from its surface value normally by orders of magnitude towards the midpart of the atmospheric boundary layer. This causes characteristically different hodograph shapes depicting coherent diurnal cycles (of heat flux and air temperature) for a homogeneous in comparison with any specialized non-homogeneous case; reference is made to Lettau (1974, Figure 3, in comparison with Figures 1 and 2). The respondance $r_H$ in Equation (6) involves the boundary value of $K$ which is known to depend on the friction velocity of the wind profile as well as the aerodynamic roughness parameter of the soil/air interface. The same factors determine the value of $\partial K/\partial z$. A positive gradient $(\partial K/\partial z > 0)$ is responsible for the fact that the phase constant by which surface temperature lags behind the $H(t)$ cycle is significantly less than $45^\circ$ (valid if $\partial K/\partial z = 0$); a useful analogy exists between $T(z, t)$-cycle hodographs and wind spirals in the atmospheric boundary layer; see Lettau and Dabberdt (1970, Figure 2) or, Lettau (1974).

For full diurnal cycles on typical soil surfaces and average wind speed, it can be expected that $r_H$ is somewhat larger than $r_G$ while $r_{G*}$ is less than $r_G$. Experimental studies to clarify the diurnal behavior of the partitioning of net radiation (Camuffo and Bernardi, 1982) should be encouraged. However, the significance of results would be enhanced if correct theoretical relationships were used in the analysis of data. This implies that direct measurements of surface temperature should be available; for appropriate radiometric methods, reference is made to Oke (1978, p. 310).

Let me demonstrate that it is most important to consider the day-versus-night differences of thermal diffusivity structure in the atmosphere. Exceptionally instructive in this respect are the detailed recordings reported by Stearns (1969) for an absolutely dry desert during a 24-hr period with strong solar forcing. Since $LE = 0$, the measurements of net radiation $(F)$, soil flux $(G)$, and near-surface temperature $(T)$ (with estimated $dT/dt$ in deg hr$^{-1}$) illustrated by Stearns (1969, Figures 7 and 9) provide the basis for the following two sets of regression equations. Note that all fluxes are converted to units of W m$^{-2}$. 