A CONDITION FOR NASH-STABILITY UNDER BINARY AND DEMOCRATIC GROUP DECISION FUNCTIONS*

ABSTRACT. A necessary and sufficient condition is established in terms of restrictions on individual preferences for a class of democratic systems to be Nash-stable: that there is no one in the society who can profitably misrepresent his preferences. It is shown that only a very strong similarity of preferences can eliminate the incentive for individuals to misreveal their preferences. Since individual preferences in actual practice are not likely to be so similar, the result in this paper shows that strategic misrevelation of preferences by individuals remains a disturbing possibility under this class of democratic rules.

INTRODUCTION

The possibility of strategic misrevelation of preferences by individuals in democratic systems has been the theme of a large number of writers in the recent literature on social choice theory. The result of the inquiry has been negative: in all interesting democratic systems, there would exist situations where individuals will find it profitable to misreveal their sincere preferences. Insofar as social decisions based on misrepresented individual preferences are not likely to have ethical appeal, this result has caused problems for ethical acceptability of democratic decisions.

The importance of this normative problem requires a search for a possible way out of the negative result. One of the possible ways to resolve the problem would be to determine the probability of having a sincere preference profile which will fail to be stable, i.e., a situation where no group of individuals can benefit by misrepresenting their preferences. If this probability is low, then, presumably, it would take the edge out of the negative result. A second possible way would be to look for the restrictions on the sincere preferences of individuals which will guarantee that, if individual preferences conform to these restrictions, then sincere preference profiles cannot be manipulated by strategic distortion of preferences. If these restrictions are ‘plausible’, in the sense that individual preferences in actual practice are likely to conform to these restrictions, then one can expect that sincere preference profiles in

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actual practice will not be disrupted by strategic voting. In this paper, we are concerned with this second line of inquiry, which has been first informally suggested by Zeckhauser [36], and which corresponds to a line of inquiry pursued in the context of Arrow's [1] General Possibility Theorem. In the context of Arrow's General Possibility Theorem, several 'restricted preference' conditions have been suggested which, if fulfilled, would ensure that certain classes of democratic systems would satisfy various 'consistency' requirements. Here also, the argument has been that if these conditions are plausible, then one need not be too worried if a system fails to be 'consistent' over a set of individual preferences which does not satisfy these conditions, for it is a set one is not likely to come across in actual practice.

Zeckhauser's [36] informal suggestion for a 'restricted preference' approach to the problem of stability was contained in his conjecture that under the familiar method of majority decision, "single-peakedness [of sincere individual preferences] is sufficient to eliminate any incentive for a voter to disguise his preferences". However, Pattanaik [23] provides a counter-example to show that single-peakedness of sincere preferences does not necessarily guarantee stability, if individuals follow any of a wide class of decision rules under uncertainty. In fact, Pattanaik's [23] counter-example shows that given a 'maximin'-type behavioural rule, even Sen's [30] 'Value Restriction' — a more general condition than single-peakedness — is not sufficient for stability of sincere preference profiles. This has raised the important question of investigating whether it is at all possible to find plausible restrictions on the sincere preferences of individuals required to ensure the stability of sincere preference profiles under democratic systems. For, if no plausible restriction can be found, then this way of escaping the normative problem will be closed.

The discussion that follows is an inquiry into this question. Assuming that the individual preferences are strong, we establish a condition on the individual preferences which is necessary and sufficient for the stability of sincere preference profiles under a large class of democratic decision procedures. However, the condition that we establish turns out to be extremely stringent, and is unlikely to be fulfilled in actual practice. The result of this paper therefore yields once more a negative conclusion: if individual preferences are strong, then, under this class of decision systems, there is no resolution of the problem of instability by postulating plausible restrictions on the sincere preferences of individuals.