STABILITY ANALYSIS OF LINEAR AND NONLINEAR PERIODIC CONVECTION IN THERMOHALINE DOUBLE-DIFFUSIVE SYSTEMS*

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Abstract

A shortcut analytic method of stability in strong nonlinear autonomous system is introduced into stability analysis of the thermohaline double-diffusive system. Using perturbation technique obtains conditions of existence and stability for linear and nonlinear periodic solutions. For linear periodic solution in infinitesimal motion, the existence range of monotonous branch and oscillatory branch are outlined. The oscillatory branch of nonlinear periodic solution in finite-amplitude motion has unstable periodic solution when \( \mu < \) critical value \( \mu_c \) in the case of \( \theta < \theta_c < \theta_s \).

The stability conclusions under different direction of vortex are drawn out.

Key words thermohaline double-diffusive system, periodic solution, stability

I. Introduction

Research of the stability problem in the thermohaline double-diffusive system focuses on stability of periodic solution. Thermohaline double-diffusive system is defined as a double-diffusive system, which include temperature and salinity. Furthermore, the temperature and salinity act as opposite function on the vertical density gradient of this system. The research of stability has important significance. Because of complexity of controlled equation, most theoreticians have neglected existence of convection term in model equation\textsuperscript{[1]} \textsuperscript{[2]}, only discussing linear periodic solution; or introducing into complex Jacobian elliptic functions method to analyze controlled equation\textsuperscript{[3]}, this method is difficult to use and to extend; or completely using numerical method to discuss\textsuperscript{[4]} \textsuperscript{[5]}. In this paper, for linear and nonlinear periodic solutions, we combine numerical method with theoretical analysis in order to obtain relatively intact results. Especially, a shortcut analytic method of stability in the analysis of nonlinear periodic solution is introduced into determining existence of solution\textsuperscript{[6]} \textsuperscript{[7]}. The method has characteristics as clear process, exact decision and easy applications.

II. Basic Equations

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We consider two-dimensional thermohaline double-diffusive system in a horizontal box of fluid confined between the sides \( z = 0 \) and \( z = H \), \( x = 0 \) and \( x = L \) (Fig. 1). Adopting the Boussinesq approximation. The temperature, salinity and density are taken to be:

\[
T^* = T_0^* + \Delta T^*(1 - z + T) \\
C^* = C_0^* + \Delta C^*(1 - z + C) \\
\rho^* = \rho_0^* (1 - \beta_1 T^* + \beta_2 C^*)
\]

where \( T^* \) is the temperature and \( C^* \) the solute concentration, and \( \beta_1, \beta_2 > 0, \Delta T^*>0, \Delta C^*>0 \). Then scaling the velocity \( \mathbf{q}^* = \frac{\kappa_T}{H} \left( \frac{\partial \varphi}{\partial z}, -\frac{\partial \varphi}{\partial z} \right) \), where \( \varphi \) is a stream function.

Scaling lengths with \( H \) and time with \( H^2 / \kappa_T \), the following dimensionless equations for \( T, C \) and \( \varphi \) are given:

\[
P_r \left[ \frac{\partial^2 \varphi}{\partial t} - J(\varphi, \nabla^2 \varphi) \right] = -R_T \frac{\partial T}{\partial x} + R_s \frac{\partial C}{\partial x} + \nabla^2 \varphi \\
\frac{\partial T}{\partial t} + \frac{\partial \varphi}{\partial x} - J(\varphi, T) = \nabla^2 T \\
\frac{\partial C}{\partial t} + \frac{\partial \varphi}{\partial x} - J(\varphi, C) = \lambda \nabla^2 C
\]

where \( J(f, g) = \frac{\partial f}{\partial x} \frac{\partial g}{\partial z} - \frac{\partial f}{\partial z} \frac{\partial g}{\partial x} \), dimensionless parameter

\[
P_r = \frac{\gamma}{\kappa_T},\ \ L_e = \frac{\kappa_e}{\kappa_T},\ \ R_T = \frac{\beta_1 g \Delta T^* H^3}{\kappa_T},\ \ R_s = \frac{\beta_2 g \Delta C^* H^3}{\kappa_T}
\]

and \( \gamma, \kappa_e, g \) are respectively the kinematic viscosity, solute diffusivity and acceleration due to gravity. Introducing into the simplest boundary conditions: at \( z = 0, 1 \), there is no normal velocity or tangential stress, and no horizontal gradients of temperature or solute concentration, while in the horizontal direction, imposing periodic boundary conditions appropriate to a cell of (dimensionless) width \( \lambda \). Thus

\[
\varphi = \frac{\partial^2 \varphi}{\partial z^2} = T = C = 0 \quad (z = 0, 1)
\]

and

\[
\varphi = \frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial T}{\partial x} = \frac{\partial C}{\partial x} = 0 \quad (x = 0, \lambda, \lambda = L/H)
\]

III. Stability of Linear Periodic Solution in Infinitesimal Motion

By linear stability theory, delete the term of \( J(f, g) \). Eqs. (2.4)–(2.6) have constant