ABSTRACT. In a previous article (see [3]) a system of axioms is proposed stating conditions which are necessary and sufficient to determine a cardinal utility function on any set, finite or infinite, of outcomes $X$. The present paper discusses and interprets the meaning of those axioms, and compares this new approach to cardinal utility with the utility differences approach proposed by Alt and Frisch, among others, and with the expected utility approach of von-Neuman and Morgenstern. The notion of repetition of the same choice situation is presented and its interpretation discussed. It is then argued that this notion leads naturally to the system of axioms presented in 'On Cardinal Utility'. It is also argued that this notion must be used if we want to have a more clear understanding of the meaning of the axioms proposed by Alt and Frisch. Finally, it is remarked that since uncertainty is not present in the new approach, it is free of the paradoxes that have plagued the expected utility hypothesis.

1. INTRODUCTION

We want to state at the outset that this is not a review article covering the extensive and important literature on the subject of Cardinal Utility or Measurable Utility, as we will, indistinctly, refer to it here. This paper is mainly concerned with discussing and clarifying the meaning of a system of axioms, leading to a cardinal utility, presented by this writer in [3], and with comparing this approach to the measurability of utility with what has been called the utility differences approach and the probabilistic or expected utility approach. Specifically, the axioms and conclusions proposed by Camacho [3] will be interpreted, analyzed and compared with the axioms and conclusions presented by Alt [1] (the utility differences approach) and with the axioms and results stated by Herstein and Milnor [5] (the probabilistic or expected utility approach).

More concretely, we will try to argue and establish the following points:

(i) that both the approach followed in [3], that we will call the repetition approach, and the utility differences approach can be based on a common fundamental assumption that we will label the repetition assumption;
(ii) that this repetition assumption will provide a more clear understanding of what we mean when we say that the difference of utilities of, say, two given outcomes is greater than, equal to, or smaller than the difference of utilities of another given pair of outcomes, thus helping to clarify one of the most debated points in the long controversy cardinal versus ordinal utility;

(iii) that the utility differences approach, although claimed otherwise in Alt's paper, provides only sufficient conditions for the existence of a cardinal utility function;

(iv) that, on the other hand, the repetition approach provides necessary and sufficient conditions for the existence of a cardinal utility function on any set (finite or infinite) of outcomes;

(v) that while topological considerations of the outcome set play a very strong role in the derivations of the utility differences approach, no restriction whatsoever is imposed on the outcome set in the derivations of the repetition approach;

(vi) that the system of axioms of the repetition approach, although apparently more cumbersome and restrictive than the system of axioms of the utility differences approach, is indeed, when properly understood, a natural consequence of the common basic repetition assumption;

(vii) that the Herstein and Milnor's results, as extended in the present paper, provide as the repetition approach also does, necessary and sufficient conditions for the existence of a cardinal utility function of any set of outcomes;

(viii) but, while the expected utility approach is based on uncertainty considerations and is subject to the well known Allais and others' paradoxes,1 both the repetition approach and the utility differences approaches are not.

The method presented in [3], as it will be made explicit below, is based on the fundamental assumption that the decision maker, when confronted with a choice situation, has the ability or power to imagine a repetition of the same choice situation as many times as he wishes. For this reason we will call this approach the repetition approach. Whether or not we accept the assumption that a choice situation can repeat itself in the real world is not an issue in the present discussion. We need only to assume that the decision maker is able to imagine such repetition.

In a previous paper [4], where, in connection with problems of social choice, we proposed essentially what we are now calling the repetition