PROVABILITY AS A DEONTIC NOTION

ABSTRACT. The purpose of this paper is to mark a significant difference between classical and several non-classical propositional calculi. The argument presupposes familiarity with Kripke/Hintikka semantics for modal logic. The non-classical systems are Hintikka’s logic of belief and alethic modal systems which have Kripke/Hintikka semantics.

The difference is marked by showing that the semantic validity operator in classical logic behaves as a normal alethic necessity-operator while the non-classical semantic validity operators behave as normal deontic ought-operators. The crucial step is showing that a formula, valid by non-classical semantics, can be falsified. I show that the negation of a non-classical thesis can be added to a consistent set of formulae without making the set inconsistent or any other set inconsistent. This is shown by observing that consistent sets of formulae do not need to be related to other consistent sets by any of the alternativeness relations of Kripke/Hintikka model structures for non-classical systems.

The deontic behavior of non-classical semantical validity operators is interpreted as showing that being a thesis of a non-classical system means, not that the thesis is a logical truth, but that the thesis is the content of a norm on how we ought to use, crucial terms such as ‘believe’ and ‘necessity’.

The purpose of this paper is to mark a significant difference between classical and non-classical proposition calculi. I begin by showing why ‘It is provable that F’ when used in Hintikka’s doxastic logic means ‘It ought to be that F’ rather than ‘It must be that F’. At the same time, it is shown that ‘It is provable that F’ when used in classical propositional logic means ‘It must be true that F’. This fact that provability in doxastic logic has the deontic sense of ‘ought to be’ while provability in classical propositional logic has the alethic sense of ‘must be’ is taken to mark the fact that Hintikka’s doxastic logic, as opposed to classical propositional logic, is not really a logic. It is not really a logic since it does not give inference rules but, at best, gives norms on how to believe. Moreover, reflection on the arguments that ‘It is provable that F’ when used in Hintikka’s doxastic system show that ‘It is provable that F’ is deontic when used in systems of alethic modal logic. This fact is taken as marking that alethic modal propositional calculi, as opposed to classical propositional logic, are not systems of logic but are systems of recommen-
dations on how to use 'necessity' and 'possibility'. After I give a sense to saying that classical propositional logic is an ethics of belief, I close with some remarks on how giving up the principle that every sentence is true or false lets 'It is provable that F', even in classical propositional logic, function as a deontic ought-operator.

At the beginning it is only fair to cite knowledge presupposed by my arguments and call attention to how I shall use some crucial terms. Familiarity with Hintikka's *Knowledge and Belief* [1] would be helpful. I am assuming familiarity with the use of Hintikka's model set and model system technique or the semantic tableaux technique for establishing whether or not a formula of an alethic modal system can be imbedded in a model structure for that system. I am also assuming familiarity with techniques of model construction so that we can say a formula has a certain kind of model if it is imbeddable in a suitable model structure. So, in effect, I am assuming familiarity with how we can establish that a formula is a thesis of a certain system by showing that its negation cannot be imbedded in any model structure for that system. For details on such sematical techniques, see Hintikka: [2], [3], [4], and Kripke: [5], [6], and [7], Hughes and Cresswell in appendix five of [8] also discuss such semantics. Capital letter F will be used to refer to formulae in particular systems. The context will make it clear what system or systems the formulae referred to are in. I shall use 'F is a thesis in system S' to mean that the negation of F, viz. ~F, cannot be imbedded in a model structure for S. I shall symbolize being a thesis of system S with ~F, where the subscript 's' will be replaced by an abbreviation of the name of a system or name of a type of system. Some abbreviations that I shall use are: cpc for 'classical propositional logic', dox for 'Hintikka's doxastic system', and ncs when I am talking of a non-classical system but no particular one. In this essay, what I mean by 'provability operators' is 'It is a thesis that F' operators. This use may seem eccentric since one may think that 'It is a theorem that F', where theorems are established by a syntactical means such as derivation from axioms, should be called provability operators. I admit that my remarks apply to 'It is a theorem that F' only when this syntactical provability operator is correlated, via a completeness proof, with a semantical provability operator of the kind I shall discuss. Here, I should emphasize that I shall talk only of propositional calculi and that by 'non-classical logic' I have in mind primarily systems of alethic modal