ABSTRACT. The puzzling coincidence of gambling and insurance has often been analysed by taking recourse to utility functions with convex and concave regions. In this paper we show that it may be optimal for utility maximizing risk seekers to engage in insurance and gambling activities simultaneously. A possible reason for this behavior is that these individuals try to take advantage of a moral hazard situation.

Keywords: Insurance, gambling, moral hazard, HARA utility.

In the economics literature it is usually assumed that individuals are either risk averters, in which case they take out insurance and avoid gambling, or risk seekers, in which case they gamble, and do not buy insurance coverage. It is common knowledge, however, that many people insure their assets, and engage in gambling activities as well. Attempts have been made to explain this puzzling behavior, either by assuming the existence of both convex and concave regions on the utility curve (Friedman and Savage, 1948; Markowitz, 1952), or by the preferred allocation of risk over time (Eden, 1977) – to name but two examples. In this paper it is shown that standard utility maximizing behavior may also imply the simultaneous purchase of insurance and engagement in gambling activities. The problem usually asked is whether it is possible that risk-averse individuals gamble. In this paper the question will be analyzed in the opposite manner, namely whether it is conceivable that risk-seeking individuals buy insurance coverage, in addition to buying lottery tickets.

Let us assume a utility-maximizing individual whose utility function is of the HARA type,

\[ U(w) = \frac{w^{1-c}}{1-c}, \]

where \( w \) stands for wealth, and \( c \) is the degree of relative risk aversion. (Positive \( c \) implies risk aversion, negative \( c \) risk seeking.) Since HARA utility functions are generally not defined for negative wealth levels,\(^2\) we consider only positive values of \( w \) in this paper.

Some part of the individual's initial wealth, \( x \), is at risk of being lost with probability \( p \). Insurance is available to protect all or part of this potential loss (denoted by \( L \)) upon payment of a premium which is computed as expected loss plus a loading,

\[
\pi = p\delta(L - D)
\]

where \( D \) is the amount left uninsured, and \( \delta \) equals one plus the loading factor. \( \delta \) is always greater than one, and, since the premium must be smaller than the amount insured, \( p\delta < 1 \). The individual must choose a level of \( D \) which maximizes expected utility:

\[
EU = pU(x - \pi - D) + (1 - p)U(x - \pi)
\]

Rewriting Equation (3) for HARA utility functions, we obtain,

\[
EU = \frac{p}{1-c} (x - \pi - D)^{1-c} + \frac{1-p}{1-c} (x - \pi)^{1-c}.
\]

We now substitute Equation (2) for \( \pi \), and take the first derivative with respect to \( D \), to obtain,

\[
\frac{dEU}{dD} = p(p\delta - 1)(x - p\delta L + D(p\delta - 1))^{-c}
+ (1 - p)p\delta(x - p\delta L + p\delta D)^{-c}.
\]

Setting this expression equal to zero, and solving, we obtain the utility maximizing amount left uninsured,\(^3\)

\[
D^* = \frac{(w - p\delta L)\Omega}{1 - p\delta \Omega},
\]

where

\[
\Omega = 1 - \left[ \frac{p\delta - 1}{p\delta - \delta} \right]^{1/c}.
\]