INDEPENDENT SOCIAL CHOICECORRESPONDENCES

ABSTRACT. A fixed agenda social choice correspondence \( \Phi \) on outcome set \( X \) maps each profile of individual preferences into a nonempty subset of \( X \). If \( \Phi \) satisfies an analogue of Arrow's independence of irrelevant alternatives condition, then either the range of \( \Phi \) contains exactly two alternatives, or else there is at most one individual whose preferences have any bearing on \( \Phi \). This is the case even if \( \Phi \) is not defined for any proper subset of \( X \).

KEY WORDS: Fixed agenda, independence, social choice

1. INTRODUCTION

A fixed agenda social choice correspondence \( \Phi \) on outcome set \( X \) maps each profile of individual preferences into a nonempty subset of \( X \). Denicolo \([7,8]\) proves that if \( \Phi \) satisfies Pareto optimality and Strong Independence (SI), a demanding version of IIA (Arrow's independence of irrelevant alternatives condition) then \( \Phi \) is dictatorial. For social choice based on a complete and transitive binary relation (which is a function of individual preferences), Campbell and Kelly \([4,5]\) established that there is almost no payoff to relaxing the Pareto criterion, however much it is weakened, if IIA remains in force. This paper makes the same point for a fixed agenda social choice correspondence. Even if Pareto optimality is discarded there is no satisfactory collective decision rule if SI is imposed: either the range of \( \Phi \) contains exactly two alternatives or else there is at most one individual whose preferences have any bearing on \( \Phi \). This follows from the independence condition alone. An important step is the demonstration that SI implies that \( \Phi \) can be generated by an Arrovian social welfare function — that is, a social welfare function having transitive values and satisfying Arrow's independence of irrelevant alternatives condition. This is true if the domain \( D \) of profiles of individual preferences for which \( \Phi \) is defined is sufficiently large. We emphasize that the feasible set, or agenda, \( X \) is fixed throughout; \( \Phi \) merely selects the 'optimal’ member of \( X \) according

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to the individual preference input. This means that we are denied two of the key axioms used by Grether and Plott [10] to extend Arrow's theorem to nonbinary social choice – Arrow's choice axiom and independence of infeasible alternatives. 1

The next section provides background, and Section 3 proves the theorem.

2. PRELIMINARIES

X is the finite set of outcomes, and N = {1, 2, ..., n} is the finite set of individuals. If Y is a subset of X let P(Y) denote the set of complete and transitive binary relations on Y. If R belongs to P(X) we let R|Y denote the relation R ∩ Y² in P(Y). D is a domain of profiles of individual preferences. That is, D is a subset of P(X)⁵, and r ∈ D specifies a preference r(i) ∈ P(X) for each i ∈ N. A strong order on X is a member ≥ of P(X) such that x ≥ y and y ≥ x both hold if and only if x = y. Let L(X) denote the family of strong orders on X. For any profile r ∈ D and Y ⊂ X, we let r|Y represent the profile q ∈ P(Y)⁵ satisfying q(i) = r(i)|Y for all i ∈ N. A social choice correspondence Φ on D associates with each r ∈ D a nonempty subset Φ(r) of X, interpreted as the set of socially best alternatives in X when individuals have the preferences specified by r. If E ⊂ D we let Φ(E) denote the set ∪r∈E Φ(r). For any r ∈ D and any Y ⊂ X we let Dr Y represent the set of r' ∈ D such that r'|Y = r|Y, abbreviated to Dr xy or Dr xyz, respectively, if Y = {x, y} or Y = {x, y, z}. The following strong independence condition was introduced by Denicolò [7]:

\[ SI \quad \text{For any } r \in D \text{ and any } x, y \in X, x \in \Phi(r) \text{ and } y \notin \Phi(r) \implies y \notin \Phi(D_{xy}^r). \]

This is a very strong condition; as Denicolò [8] points out, it is not satisfied by the Pareto optimality correspondence. Alternative x may be Pareto undominated at r and y may be Pareto dominated, by z but not by x. We can have another profile, r', agreeing with r on \{x, y\} as far as the individual orderings are concerned, but with y Pareto undominated and with z Pareto dominating x at r'. (Sen’s Independent Decisiveness condition, introduced in Sen [12], is weaker than SI, but is not satisfied by the Pareto optimality correspondence.