Instability Finite Element Analysis of Fibre Reinforced Composite Structures Based on the Third Order Theory

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Abstract. In this paper a refined higher-order shear deformation theory for the instability finite element analysis of fibre reinforced shell like composite structures is developed. A higher order shear deformation theory allows parabolic description of the transverse shear stresses and therefore the shear correction factors of the usual shear deformation theory are not required. The present formulation is based on a higher order shear theory in which in-plane displacements are expanded as cubic functions of the thickness coordinate. The conditions of zero transverse shear stresses on the top and bottom faces are satisfied. Laminate material is assumed to be linearly elastic, homogeneous and isotropic/orthotropic. The 4-node quadrilateral shell finite element with 8 degrees of freedom per node has been developed which eleviates most of the deficiencies associated with such types elements. The effects of the transverse shear deformation on the buckling loads are investigated. It is shown that the present theory predicts buckling loads more accurately then CTP or the first order shear deformation theory. To determine buckling loads here is used Lanczos algorithm. The good agreement of the numerical and experimental results is indicative of a reliable presented shell element and procedure for practical instability analysis of the structures made of the fiber reinforced laminates.

Key words: buckling, layered shells, finite elements, eigen problem, Lanczos algorithm.

1. Introduction

In recent years, the interest of the aerospace industries has been directed toward the multilayered fibre-reinforced composite structures for their high modulus, high strength, and low weight [7]. The capability to predict the buckling behavior of anisotropic and laminated CFC shell structures, when subjected to the compressed loads is of prime interest to structural analysis. It is well-known from experimental observations that the Poisson–Kirchhoff theory of plates underpredicts reflections and overpredicts natural frequencies and buckling loads. These results are due to the neglect of transverse shear strains in the classical plate theory (CPT). Improvements of the classical plate theory, based on Kirchhoff's assumptions, have been proposed by many researches [1, 2]. First order, Reissner–Mindlin plate theories (FOST), including the effect of the transverse shear deformation on the buckling behavior of laminated composite plates, have been used by a great number of investigators [3, 4]. In Refs. [5–7] higher-order theories (HOST) have also been developed. The classical laminate plate theory is adequate
for many engineering problems. However, laminated shells made of advanced filamentary composite materials, whose elastic to shear modulus ratios are very large, are susceptible to thickness effects because their effective transverse shear moduli are significantly smaller than the effective elastic moduli along fiber directions. These high ratios of elastic to shear modulus render the CPT inadequate for the buckling analysis of composite shells. The present effort focuses on the development of the finite element models to implement in the instability analysis of thin to thick shell type composite structures. Since the higher-order shear deformation theory gives as accurate a global response as the 3-D theory but is computationally less demanding, it is considered efficient for buckling problems. The HOST plate theories provide improved global response estimates for buckling loads. A 4-node shell element is derived combining HOST and Discrete Kirchhoff’s Theory (DKT) [11, 12]. An special attention in this paper is focused on the buckling behavior of the laminated panels with centrally located circular holes subject to axially compressed load. Good agreement of the present finite element solutions with the experimental results was obtained.

2. Governing Equation

Composite laminates are modeled using one of the following three classes of theories: i) equivalent single-layer 2-D theories, ii) layer-wise 2-D theories, and iii) continuum-based 3-D and 2-D theories. Of the three classes, the single-layer displacement theories are the simplest and most economical to use. The choice of an appropriate theory, the manner in which transverse shear effects are included, and the advantages of that theory and finite element model relative to computational cost are critical issues. Thus, in large-scale applications the elements of choice are those that provide the best compromise between accuracy and computational cost, with the displacement based theories emerging as the preferred framework. Formulation of the presented shell finite element is based on the single-layer 2-D theory because it is adequate in representing global behaviour (e.g. deflections, stresses, buckling loads) of thin composites. In this section, a brief presentation of the governing equations of motion corresponding to the present shear deformation theory is given as

\[(K + \lambda K_G)\Phi = 0\]  \hspace{1cm} (2.1)

where \(K\) and \(K_G\) are the global stiffness and geometric stiffness matrices respectively (obtained by the assembly of the corresponding element matrices), \(\lambda\) is the eigenvalue and \(\Phi\) is the buckling mode. Lanczos algorithm [8, 5] is used to obtain the eigenvalues \(\lambda_i\) and the corresponding eigenvectors \(\Phi_i\) (see Appendix A). The HOST used here, takes into account the parabolic distribution of the transverse shear stress along the laminate thickness. The displacement field for