Abstract. Abraham Lincoln's dictum that "you may fool all of the people some of the time; and some of the people all of the time; but you can't fool all of the people all of the time", is interpreted in terms of a simple binomial model, and potential ambiguities in Lincoln's assertion are clarified.

Let us consider Abraham Lincoln's famous dictum in the light of a simple combinatorial model. Consider a group of size $N$ of mean competence $p$, where $(1-p)$ is taken to be the likelihood that an average individual will be fooled on any given occasion. What's the probability that you can fool all of the people some of the time? Let us interpret some of the time the way logicians do, as meaning at least once. The probability that everyone is fooled on any given occasion is simply $(1-p)^N$. Even if $p$ is very small, $(1-p)^N$ rapidly goes to zero as $N$ gets larger. Thus, on any given occasion it is unlikely that everyone will be fooled. On the other hand, if we (simultaneously) confront the members of a group with a sufficiently large number of chances to be fooled ($K$ times, say) then it's likely that on at least one of these occasions they will be fooled.

The probability that everyone in a group will be fooled at least once in $K$ times is $1 - (1 - (1-p)^N)^K$. Regardless of $p$, for fixed $N$, this expression goes to one as $K \to \infty$; for fixed $K$, this expression goes to zero as $N \to \infty$. A natural question to ask is what happens when $N$ and $K$ simultaneously increase. Regardless of $p$, if $K = N$, the above expression goes to zero as $N$ and $K$ increase toward infinity. Even if there are considerably more
opportunities to fool people than there are people to be fooled, as long as \( N \) rises proportional to \( K \) (i.e., \( N = \alpha K, 0 < \alpha < 1 \)), the probability of fooling all the people some of the time still goes to \textit{zero} as \( N \to \infty \), regardless of \( p \). Thus \textit{it is not} as easy to fool all the people at least some of the time as we (or Lincoln) might think.

Of course, if we interpret "fooling all of the people some of the time" as "fooling everybody at least once, but not necessarily at the same time"; then, under our assumptions, the probability of such an event may be expressed as \((1 - p^K)^N\). Thus

\[
\lim_{\substack{N \to \infty \\ K \to \infty}} (1 - p^K)^N \to 1, \quad \text{for all } p.
\]

Fooling everybody at least once, but not necessarily at the same time is considerably easier than fooling everybody at the same time at least once. Moreover, the former gets easier to do as \( N \) and \( K \) simultaneously increase; the latter gets increasingly more difficult!

Now, let us consider the case of fooling some of the people all of the time. The probability that any given member of the group will be fooled \( K \) times in a row is simply \((1 - p^K)^N\). As \( K \) gets larger, this probability rapidly goes to zero. Let us interpret "some people" as "at least one person". The probability that some set of one or more persons in a group of size \( N \) will be fooled \( K \) times in a row is simply

\[
\sum_{h=1}^{N} \binom{N}{h} (1 - p^K)^h (1 - (1 - p^K)^{N-h})
\]

This may be rewritten as

\[
1 - \binom{N}{0} (1 - (1 - p^K)^N) = 1 - (1 - (1 - p^K)^N).
\]

This expression is equivalent to that obtained earlier for the probability of fooling all the people some of the time, except that \( K \) and \( N \) are interchanged. This interchange does not affect the behavior of the expression in the limiting case. Thus, similarly,

\[
\lim_{\substack{N \to \infty \\ K \to \infty}} 1 - (1 - (1 - p^K)^N \to 0, \quad \text{for all } p.
\]