A NEW CONCEPT OF VERISIMILITUDE

0. SUMMARY

Popper's verisimilitude is the excess of truth content over falsity content. It is shown that his measures of truth and falsity content are at variance with his respective concepts. It is further shown that both his actual measure of verisimilitude and measures based on measures of truth and falsity content consistent with his definition of the concepts, have undesirable properties. Moreover, any measure of verisimilitude based solely on content and truth value does not capture the notion of closeness to truth. A new concept of verisimilitude is proposed, based on a metric in the space of state descriptions.

1. STATE DESCRIPTIONS, RANGE AND CONTENT, AND LOGICAL PROBABILITY

We define a state description (SD) as the strongest (most informative) consistent statement in a language system. It suffices here to consider the simplest example of a language system, a sentential calculus with n primitive sentences. A SD then is a conjunction of the n primitive sentences, each of which may or may not be preceded by a negation sign. There are $N = 2^n$ SDs.

Any two SDs are incompatible. Given a statement $a$, there is the set of SDs with which it is compatible, its range; and the set of SDs with which it is incompatible, its content. A statement is logically equivalent to the disjunction of the SDs in its range. The range of a tautology is the universal set, its content the empty set; the range of a contradiction is the empty set, its content the universal set.

Logically equivalent statements have the same range and the same content. In this article we consider statements as represented by their range or content, i.e. we consider logically equivalent statements as identical, we do not care for their formulation.

Exactly one SD in the language system is true, for which we write 't'. A statement is true if $t$ is in its range.

A measure of the range of a statement $a$ is a logical probability $p(a)$;
1 -- $p(a) = Ct(a)$ then is the corresponding content measure. In this article we define

$$p(a) = \frac{N_a}{N}$$

where $N_a$ is the number of $SD$s in the range of $a$, and $N$ is, as above, the total number of $SD$s in the language system.

2. POPPER’S CONCEPTS OF CONTENT, TRUTH CONTENT AND FALSBITY CONTENT

The content$_p$ (to be distinguished from the content defined in section 1) of a statement $a$ is defined by Popper (1963, p. 218) as “the class of all those statements which are logically entailed by it”. The truth content of $a$ is “the class of the true logical consequences of $a$”, and the falsity content of $a$ is “the class of the false consequences of $a$” (p. 233). So the content$_p$ is subdivided into two mutually exclusive and exhaustive subclasses, the truth content and the falsity content. Since a true statement has no false consequences, its falsity content is empty, and its truth content is its content$_p$.

3. POPPER’S MEASURES OF CONTENT, TRUTH CONTENT, FALSITY CONTENT, AND VERISIMILITUDE

3.1. The measure of content

As a measure of content Popper (1963, p. 392) defines

$$Ct(a) = 1 - p(a)$$

in accordance with the $Ct$ defined here in section 1, which is related to content; but how is $Ct$ related to content$_p$, the set of consequences?

We can easily characterize the set of all consequences of $a$. Any consequence of $a$ is generated from it by adjoining to its range some subset of the $SD$s in its content, which contains $N - N_a$ $SD$s, so that there are $2^{N-N_a} = c_a$ possibilities of doing so. I.e., there are $c_a$ consequences of $a$, including $a$ itself (generated by adjoining the empty subset of the $SD$s in content($a$)). Using the definition of $p(a)$ from section 1, we have:

$$Ct(a) = 1 - p(a) = 1 - \frac{N_a}{N} = \frac{N - N_a}{N} = \frac{1}{N} \log_2 c_a.$$