REMARKS ON SOME RECENT PAPERS CONCERNING LINE FORMATION IN A MAGNETIC FIELD

(Research Note)

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Since several years a series of papers appears which deals with different aspects of line formation in a magnetic field. Different symbols used by several authors for the same physical quantity, but also the use of the same symbol for various quantities may easily confuse and result in errors. Thus e.g. the expressions for anomalous dispersion in the absorption matrix of the equations of transfer given by Beckers (1969a, b) differ from those of Rachkovsky (1962a, b, 1967, 1969) by a factor \((2 H(a,0))^\nu\). (\(H(a, v)\) is the Voigt function, \(a\) the damping and \(v\) the distance from the line centre, both in units of the Doppler width). This factor has been taken over into other papers, e.g. by Göhring (1970) and in the numerical calculations in Section 3C2 of our earlier paper (Staude, 1970b); for example Figure 1 shows the influence

Fig. 1. Calculated line contours \(r_\nu\) for Fe I 5250.2 Å, photospheric model BCA, position \(\mu = 1\); magnetic field \(H = \pm 2000\) G, inclination \(\Psi = 75^\circ\), azimuth \(\Phi = e^{-10} \tau\); the central (dash-dot) curve without anomalous dispersion, the outer curves for exact expressions of anomalous dispersion, the dashed curves for the expressions of Beckers (1969a, b).
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Of the wrong factor on line contours \( r_r \) (corresponding to Figure (6a) in our earlier paper). In his clear review paper, which also explains different formulations of line formation in a magnetic field, Stenflo (1971) has eliminated the wrong factor \( \frac{1}{2} \) from the expressions of Beckers but he has taken over the wrong \( 1/H (a, 0) \). This results from the fact that in earlier papers, e.g. according to ten Bruggencate et al. (1955) and Kjeldseth Moe (1968), \( \eta_0 \) is defined as the ratio between the line absorption coefficient \( \kappa_c \) in the line centre for zero damping and the continuous absorption coefficient \( \kappa \). This definition is used generally in other quantities like \( \eta_r, \eta_Q \) and \( \eta_V \) or \( \eta_p, \eta_l \) and \( \eta_r \) (Unno, 1956; Stenflo, 1971, Equations (20)). When deriving his expressions for anomalous dispersion Beckers (1969a) defines in Equation (5) the line absorption coefficient \( \kappa'_c \) taking into account damping and thus the factor \( H (a, 0) \), but in the resulting Equations (10, 13, 14) he uses the old symbol \( \eta_0 \) for \( \kappa'_c/\kappa \). Two further differences between the papers of Rachkovsky and those of other authors (Beckers, 1969a, b; Göhring, 1970; Stenflo, 1971) should be mentioned: The expressions of Faraday rotation \( q_r \) are opposite in sign in the two cases; moreover the azimuth of the magnetic field \( H \) is defined by Rachkovsky (1969) 90° different from that by the other authors.

In a recent paper Rees (1971) criticizes the absorption matrix in our earlier paper (Staude, 1969; Equation (5b)) and declares it to be wrong. In our opinion this criticism is justified only inasmuch as in this paper the meaning of the matrix has not been explained distinctly enough; the main purpose of Section 3 was to derive the general solution (Equation (9)) which doesn’t depend on the special form of the absorption matrix. A clear interpretation of the matrix and also the relation to the representation of Beckers (1969a, Equations (1)) are given, however, in connection with numerical calculations in another paper (Staude, 1970b, Section 3A): The equations of Unno (1956; valid for zero azimuth of \( H \) and a coordinate system fixed in space) are generalized in our matrix (5b) for arbitrary depth dependence of the coordinate system (plane \( X-Y \)) in which the Stokes parameters \( Q \) and \( U \) are defined (\( I, \sqrt{(Q^2+U^2)} \) and \( V \) are invariant). The model calculations carried out by Rees for \( \gamma = \pi/2 \) (not \( \pi \) as written by him), \( \chi = 0^\circ \) (‘Unno’) and \( \chi = e^{-10r} \) (‘Beckers’ and ‘Staude’) refer to three different cases: For ‘Beckers’ \( \chi (\tau) \) is the azimuth of \( H \) in a coordinate system fixed in space, but for ‘Staude’ \( \chi (\tau) \) is the azimuth of a variable coordinate system. Therefore the results for ‘Unno’ and ‘Staude’ must coincide for \( \sqrt{(Q^2+U^2)} \) and \( I \) (the difference of the latter in Figure 3 of Rees may only be caused by inaccuracy of his numerical computations)*; coincidence for \( Q \) and \( U \) is also obtained if both coordinate systems coincide at \( \tau \approx 0 \) (in Rees’ calculations they differ by \( \Delta \chi \approx 1 \)). Thus the remarks of Rees on our matrix (5b) are wrong because they impune an erroneous interpretation.

The equations of transfer in a non-uniform magnetic field taking into account scattering and pure absorption were first derived in a very interesting paper by

* After completion of the present paper Mr. Rees has informed me by letter that his computer program used to calculate Figure 3 contains an error (a factor 2 was omitted from the \( d\chi/d\tau \) terms) which slightly affects \( I \) and \( U \) and inverts the sign of \( Q \).