A MODEL OF THE MAGNETIZED SOLAR WIND

I. H. URCH
Dept. of Physics, University of Adelaide

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Abstract. A steady state, inviscid, single fluid model of the solar wind in the equatorial plane is developed using magneto-hydrodynamics and including the heat equation with thermal conduction but no non-thermal heating (i.e. a conduction model). The effects of solar rotation and magnetic field are included enabling both radial and azimuthal components of the velocity and magnetic fields to be found in a conduction model for the first time.

The magnetic field cuts off the thermal conduction far from the sun and leads to an increased temperature at 1 AU and relatively small changes to the radial velocity and density. Models have been found which fit the experimental electron densities in 2 $R_O < r < 16 R_O$. These models predict at 1 AU a radial velocity of 300–380 km·sec$^{-1}$ and a density of $\approx 8$ protons·cm$^{-3}$. The latter velocity corresponds to a density profile obtained by Blackwell and Petford (1966) during the last sunspot minimum, and is about 100 km·sec$^{-1}$ above that found in previous conduction models which fit the coronal electron densities. The radial velocities are now consistent with the mean quiet solar wind, as are the densities when the experimental values are averaged over a magnetic sector. However, the azimuthal velocity at 1 AU is only 1–2 km·sec$^{-1}$ which is low compared to the experimental values, as found by previous authors.

1. Introduction

Satellite measurements (see, e.g. the review article by Axford (1968)) have established the existence of a continuous solar wind. They confirm the conclusions of Parker (1958) where he showed that a static solar corona would be unstable, and instead it would expand into a supersonic wind at large heliocentric radii, $r$, from the sun. Following this pioneering paper the solar wind flow has been the subject of extensive theoretical analysis using the hydrodynamic conservation equations described in Section 2. At first a polytropic representation for the temperature was used and the heat equation neglected, e.g. Parker (1963). Chamberlain (1961) assumed that there is no non-thermal heating above the base of the corona and was then able to include the heat equation. His analysis gave rise to the 'solar breeze' because he chose a solution where there was no net energy carried away by the expanding plasma. Discussions followed by Noble and Scarf (1963); Scarf and Noble (1965); Parker (1964b); Whang and Chang (1965); Whang et al. (1966); Hartle and Sturrock (1968) where the heat equation was included to obtain supersonic solutions corresponding to the solar wind.

They all assume steady state, spherically symmetric and radial flow. Consequently complicating effects caused by the magnetic field structure and solar rotation have been neglected. Weber and Davis (1967) have included these latter effects but in a polytrope model. In a conduction model the magnetic field has an additional effect. Namely, the thermal conduction takes place only along the lines of force. This leads to the effective thermal conductivity being greatly reduced at large distances from the sun.

In this paper a single fluid inviscid, steady state conduction model is developed for
the flow of the solar wind in the equatorial plane, where the presence of solar rotation and magnetic field are considered. It is further assumed that there is axial symmetry about the sun's rotation axis and that the magnetic field has $B_\phi = \partial B_\theta / \partial \theta = 0$ on the equatorial plane, where $(R, \theta, \phi)$ are spherical polar co-ordinates (defined about the rotation axis). Under these assumptions the solar wind temperature, density, velocity and magnetic fields are calculated, and compared with the real solar wind.

The equations are set up in Section 2 and the boundary conditions discussed in Section 3. In Section 4 the numerical method used to integrate the equations is briefly discussed. The numerical results are then presented in Section 5 and discussed in Section 6.

2. Basic Equations

To calculate the solar wind parameters, temperature $T$, velocity $v$, total number density $n$, and magnetic field $B$, we can use the hydromagnetic equations of conservation of mass, momentum and energy together with Maxwell's equations. Considering a steady state, inviscid expansion with no heat sinks or sources and a maxwellian velocity distribution for the ions and electrons then the conservation of mass, momentum and energy may be expressed by

$$\nabla \cdot (nv) = 0$$  \hspace{1cm} (2.1)

$$\langle m \rangle \ n v \cdot v = - \nabla (nkT) + \langle m \rangle \ n F + J \times B$$  \hspace{1cm} (2.2)

$$\nabla \cdot (\rho_v \ n v^2 v + \frac{3}{2} nkT v - \kappa \cdot \nabla T) = \langle m \rangle \ n F \cdot v + J \cdot E$$  \hspace{1cm} (2.3)

where $\langle m \rangle$ is the mean mass per particle, $\kappa$ is the thermal conductivity tensor, $F$ is the gravitational force per unit mass and the e.m.u. system of units is used.

In previous models employing the energy equation the magnetic field has been neglected, resulting in the thermal conductivity being considered as a scalar. In a strong magnetic field, $\omega_p \tau_p \gg 1$, ($\omega_p$ is the proton cyclotron frequency and $\tau_p$ the proton collision time) the thermal conduction flux may be expressed as

$$\Gamma_{\text{thermal}} = - \kappa'' (VT)^{''} - \kappa^\perp (VT)^\perp$$  \hspace{1cm} (2.4)

where $''$ and $\perp$ superscripts indicate parallel and perpendicular directions to the magnetic field. Braginskii (1965) gives

$$\kappa^\perp = \kappa'' / (\omega_p \tau_p)^2$$  \hspace{1cm} (2.5)

$$\omega_p \tau_p \approx 10^4 BT^{3/2} / n_e$$  \hspace{1cm} (2.6)

where $\kappa''$ is the non-magnetic thermal conductivity. At the orbit of the earth $T \approx 10^5$ K; $n_e \approx 8$ electrons·cm$^{-3}$ and $B \approx 5\gamma$ yielding $\omega_p \tau_p \approx 2.0 \times 10^6$, while in the low corona $n_e \approx 10^8$ electrons·cm$^{-3}$, $T \approx 10^6$ K and $B \approx 2$ G, yielding $\omega_p \tau_p \approx 2.0 \times 10^5$. Hence the perpendicular component of the thermal conduction flux can be neglected. Scarf and Noble (1965) have shown that a 10% helium content has negligible effect upon the thermal conductivity and so Spitzer’s (1962) value for a pure hydrogen plasma can