THE RELEVANCE OF THE BALLOONING APPROXIMATION
FOR MAGNETIC, THERMAL, AND COALESCED
MAGNETOTHERMAL INSTABILITIES

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Abstract. Approximate solutions of the linearized non-adiabatic MHD equations, obtained using the ballooning method, are compared with 'exact' numerical solutions of the full equations (including the effects of optically thin plasma radiation). It is shown that the standard ballooning method, developed within the framework of ideal linear MHD, can be generalized to non-ideal linear MHD. The localized (ballooning) spectrum has to be used with caution, but can give valuable (though limited) information on non-ideal stability.

The numerical analysis also confirms and quantifies the interesting connection between magnetic and thermal instabilities. The existence of such a coupling is inherent in many qualitative discussions of magnetic disruptions. Finally, the hitherto unrecognized role of the thermal continuum in the unstable part of the 'magnetothermal' spectrum is investigated.

1. Introduction

The ballooning method is well-documented in ideal MHD. It has in the first instance been applied successfully to investigations of pressure-driven ballooning instabilities in an ideal toroidal plasma (Connor, Hastie, and Taylor, 1979), whence its name. A detailed and rigorous mathematical treatment was constructed by Dewar and Glasser (1983). These authors also strongly simplified the search for the most unstable global mode. It was soon recognized that the ballooning method has a wider validity than the original domain of application. The method was consecutively applied to solutions which are not 'ballooning modes' in the original, pictorial sense of the term, but have the essential characteristics: a slow variation along the magnetic field, but a rapid variation across the field lines in the magnetic flux surfaces. The ballooning method turned out to be very successful in various branches of plasma physics. In solar physics, it was applied to facilitate the inclusion of the effects of magnetic field line anchoring in the photosphere, called line-tying (Hood, 1986). Even for a one-dimensional equilibrium, this line-tying makes the linear analysis intrinsically two-dimensional. The ballooning approximation reduces the problem back to a one-dimensional analysis for solutions localized in the flux surfaces.

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The mathematical treatment by Dewar and Glasser (1983) and the first applications were limited to linear ideal MHD. The ease of use of the ballooning method afterwards encouraged physicists to use the method in non-ideal MHD as well. Velli and Hood (1986, 1987) studied resistive instabilities in solar coronal loop and arcade equilibria close to marginal stability using the ballooning method. Their analysis was then extended to visco-resistive modes by Van der Linden, Goossens, and Hood (1988), who studied the combined effects of resistivity and parallel and perpendicular viscosity for coronal arcades. The general equations for ballooning modes, including the above effects, as well as thermal conduction, gravity and plasma radiation, were derived by Hood, Van der Linden, and Goossens (1989), who also addressed the problem of the correct line-tying boundary conditions. Hood et al. (1992) and Cargill and Hood (1992) then used these equations (neglecting resistivity, viscosity, and gravity) to investigate thermal instabilities in coronal arcades, respectively, without and with line-tying.

All the above mentioned non-ideal MHD results were evaluated in the assumption that the methods from ideal MHD apply for non-ideal MHD also, but so far there has been no assessment of the true relevance of these results. Specifically for the type of solutions obtained by Hood et al. (1992) and Cargill and Hood (1992) this appears to be a major shortcoming. Clearly, the solutions on one individual flux surface have no physical significance, but the basic idea behind the ballooning method is that by examining the solutions on all flux surfaces simultaneously, realistic physical information can be obtained. Unfortunately, the analysis of Dewar and Glasser (1983) in ideal MHD cannot easily be generalized to non-ideal MHD. Therefore, we have first compared the ballooning solutions with numerical solutions of the full equations. These numerical solutions were obtained using the powerful finite element code LEDA (for a description of the finite element method see Kerner et al., 1985). The numerical calculations also enabled us to study the unexpectedly strong interaction of magnetic and thermal modes, including the role of the thermal continuum herein. This interaction can have significance for the study of apparently magnetic disruptions such as solar flares. Already, it is often mentioned that it may be important to include thermal instability effects in the description of, e.g., solar flares. This onset process appears to be basically a magnetic phenomenon (including reconnection, so resistivity may be important as well), but obviously thermal conduction and radiation effects cannot be neglected. Indeed, if there is sufficient coupling of thermal to magnetic perturbations, a thermal instability may turn out to be the essential trigger to many disruptive phenomena.

The existence of such a coupling is suggested by Hood et al. (1992). Their results, obtained using the ballooning approximation, include the finding that on a number of magnetic flux surfaces, the local ballooning solutions for thermal and magnetic instabilities coalesce to form a complex conjugate pair of solutions. However, since it is not obvious how these infinitely localized modes relate to physical global modes, the picture obtained from the ballooning method can be at best qualitatively correct. It still needs to be demonstrated that also in a global approach coalesced 'magnetothermal' modes exist, and if so, how these are related to their localized counterparts. Also, the localized