SIMULATING PHOTOSPHERIC DOPPLER VELOCITY FIELDS

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Abstract. A method is described for constructing artificial data that realistically simulate photospheric velocity fields. The velocity fields include rotation, differential rotation, meridional circulation, giant cell convection, supergranulation, convective limb shift, p-mode oscillations, and observer motion. Data constructed by this method can be used for testing algorithms designed to extract and analyze these velocity fields in real Doppler velocity data.

1. Introduction

Artificial data play a crucial role in testing analysis techniques in many scientific disciplines. These data have known characteristics which should produce predictable results from the analysis technique. On the one hand, the artificial data can be extremely simple, so that only one aspect of the analysis is tested. On the other hand, however, some data must be realistic enough to closely match the characteristics of the natural phenomena under investigation.

Artificial data have been used before to simulate Doppler velocity measurements of photospheric flows. For example, Christensen-Dalsgaard (1984) and Balandin, Grigoryev, and Demidov (1987) used a representation of the global oscillation velocities to construct spatial filters for isolating different modes of oscillation. Hathaway (1987) constructed simple velocity fields representing the steady photospheric flows to test a technique for isolating different modes of convection and the large-scale steady flows. However, this earlier work did not allow for a tilt of the Sun’s rotation axis toward the observer, did not include components due to the p-mode oscillations or the motion of the observer, used the velocity at pixel center rather than an average over the pixel, and did not include a dense and broad spectrum to realistically represent the convective motions such as supergranulation.

With the anticipation of moderate and high resolution Doppler images from the Global Oscillation Network Group (GONG) and the Solar Oscillation Imager (SOI) on the Solar and Heliospheric Observatory (SOHO), artificial data with similar resolution will need to be constructed for testing analysis routines and procedures. A method for producing such data is described in this paper. Section 2 gives an overview of the method. Section 3 provides the practical details for constructing the images and Section 4 presents a candidate spectrum and the resulting images for a realistic simulation of the photospheric motions.

Single Doppler images, like those produced here, can be used to test various algorithms devised for identifying the modes of oscillation or different components of the steady flow. Methods for removing the results of atmospheric seeing and scattering...
or various instrumental effects can be tested by introducing these effects in the images and then comparing corrected images with the originals. A time series of these images could simulate the actual data strings from the GONG or SOI instruments and would be useful for testing techniques for filling data gaps, for merging simultaneous data from different sites, and for testing temporal filters for separating the \( p \)-mode signal from the steady flow signal. Images produced by the method described below are needed as the first step in many such studies. They represent the pixel averaged velocity as seen by an ideal instrument above the Earth’s atmosphere.

2. A Method of Producing Artificial Doppler Images

The vector velocity field on the surface of a sphere can be represented by a spectrum of poloidal and toroidal modes (Chandrasekhar, 1961) with

\[
V_r(\theta, \phi) = \sum_{l=0}^{l_{\text{max}}} \sum_{m=0}^{l} R_l^m Y_l^m(\theta, \phi),
\]

\[
V_\theta(\theta, \phi) = \sum_{l=1}^{l_{\text{max}}} \sum_{m=0}^{l} \left[ S_l^m \frac{\partial Y_l^m(\theta, \phi)}{\partial \theta} + T_l^m \frac{1}{\sin \theta} \frac{\partial Y_l^m(\theta, \phi)}{\partial \phi} \right],
\]

\[
V_\phi(\theta, \phi) = \sum_{l=1}^{l_{\text{max}}} \sum_{m=0}^{l} \left[ S_l^m \frac{1}{\sin \theta} \frac{\partial Y_l^m(\theta, \phi)}{\partial \phi} - T_l^m \frac{\partial Y_l^m(\theta, \phi)}{\partial \theta} \right],
\]

where \( Y_l^m(\theta, \phi) \) is a spherical-harmonic function of degree \( l \) and azimuthal order \( m \), \( \theta \) is the colatitude measured southward from the north pole, and \( \phi \) is the azimuth measured prograde from the central meridian. The complex quantities \( R_l^m, S_l^m, \) and \( T_l^m \) are the spectral coefficients for the radial, poloidal, and toroidal components, respectively. These coefficients are coupled by the equations of motion for the fluid. Much of the physics to be learned in analyzing the data is concerned with the nature of this coupling and the magnitudes of the various coefficients. Some of the advantages of using this spherical harmonic representation are presented in Appendix A.

To simulate the observed line-of-sight velocity, the spectral coefficients in (1) are specified and the three vector velocity components are calculated and then projected onto the line-of-sight. The result must then be integrated over a picture element (pixel) to simulate the acquisition of Doppler data. The line-of-sight velocity at a point \((\theta, \phi)\) is given with sufficient accuracy by

\[
V_{\text{los}}(\theta, \phi) = V_r(\theta, \phi) \left[ \sin B_0 \cos \theta + \cos B_0 \sin \theta \cos \phi \right] +
\]

\[
+ V_\theta(\theta, \phi) \left[ \sin B_0 \sin \theta - \cos B_0 \cos \theta \cos \phi \right] +
\]

\[
+ V_\phi(\theta, \phi) \left[ \cos B_0 \sin \phi \right], \tag{2}
\]

where \( B_0 \) is the latitude at disk center (or equivalently the tilt of the Sun’s north pole toward the observer) and velocities away from the observer are taken to be positive.