THE INTERPRETATION OF TOTAL LINE INTENSITIES FROM OPTICALLY THIN GASES

I. A General Method

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Abstract. We describe a general method for inferring, from the line emission of an optically thin medium, the physical state of the gas along the column in the line of sight which is sampled by the observations. Since it is not possible to infer the distribution of the physical state parameters with position in the line of sight – any arbitrary rearrangement of material giving equivalent line emission – we seek instead to specify the state in another way. A unique specification is found in terms of the bivariate distribution function $\mu(n, T)$, describing the partitioning of the matter in the gas over the density and temperature. We show that, given sufficient observational data, it is in principle possible to determine both $\mu(n, T)$, and the chemical composition. With less complete data the acuity of the analysis is correspondingly reduced.

The method is devised for application to the astronomical case, especially for studies of the solar corona, the chromosphere-corona transition region, planetary nebulae and other optically thin sources. We illustrate the formulation for the situation encountered in the solar corona.

1. Introduction

In astronomy, as in laboratory physics, the spectral line emission from optically thin gases frequently forms the basic data from which to infer the physical conditions in the source. Instances are many, we mention simply the problems of interpreting the line spectrum of the solar corona, of the chromosphere-corona transition region, and the planetary nebulae, all of which are optically thin to most if not all of the radiation produced by the gas.

If the opacity of the gas is low in a certain line of interest, and to all radiation playing a significant part in populating the upper state of the line, then there will be nothing in the emitted line, in its profile or intensity, to serve as a tool for determining the dependence of the physical properties of the gas on position along the line of sight. Any simple geometrical rearrangement of the material will produce the same

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emission in all optically thin lines so that all such complexions of the gas will be indistinguishable in terms of the emission in these spectral lines. Correspondingly a general technique for the spectroscopic diagnosis of an optically thin gas must be formulated in such a way that we seek to determine only those properties of the gas which are preserved under a geometrical rearrangement of the emitting material in the line of sight. A central point of this paper is to obtain such a formulation.

Of course, it is possible to determine the state of the gas at each point if one is prepared to make some geometrical assumption to relate conditions along the direction of observation to those at right angles to it. Thus, for optically thin laboratory discharges one is frequently justified in assuming cylindrical symmetry, the experiments being designed with this in mind. In such a case the local value of the emissivity in any particular line can be determined through a simple inversion of an Abelian integral equation – a standard technique widely used in laboratory studies (see e.g. Pearce, 1958). The method has also been applied to astrophysical situations where the variation of the emission with position in the sky plane suggests it to be valid. Thus, for example, the Abelian inversion method has been applied to a prominence and limb flare by Jefferies and Orrall (1961) and to coronal condensations by Aly et al. (1962). However, the measured distribution of intensity across the source can be used to find a volume emissivity at each point only if we make an assumption of geometrical symmetry. A coronal condensation, a solar prominence, or a nebula may show complex filamentary structure on photographic plates, however, and one has no basis in such cases for an assumption of symmetry, unless it is for the very finest features.

For a transparent gas, the observed total intensity in a spectral line \( I \), is the integral of the emissivity \( E \) over the line of sight, i.e.,

\[
I = \int_0^\infty E(x) \, dx = \int_0^\infty E [T(x), n(x)] \, dx.
\]  

(1)

The functional dependence of \( E \) on electron density \( n \), and temperature \( T \), is known reasonably well from theory and in general differs significantly from line to line – a fact that will be central to any diagnostic scheme*. Our aim, as stated above, will be to cast Equation (1) into a form free of the variable \( x \), since \( n(x) \) and \( T(x) \) are not uniquely determinable for an optically thin gas. We shall find it useful to introduce distribution functions to describe the partitioning of the material, for example among the different temperatures present in the gas, rather than among the line of sight position \( x \) since the former may be common to all states of the gas having the same spectral line emission, whereas the latter is certainly not.

* We assume implicitly that the emissivity in a line (integrated over the line shape) is controlled by the electron density and temperature, along with other factors whose values are independent of the distribution of material in space. If this is not so, for example if neutral atoms controlled the gas excitation, then some obvious modifications to the formulation would be necessary. In the astronomical applications we have in mind, the limitation of the state parameters to electron density and temperature and chemical composition will normally suffice.