ABSTRACT. The paper deals with the characterization of a class of social welfare orderings. The social evaluation functions which represent these orderings are separable in the components of the ordered utility vector. The characterization is based on the Strong Pareto Property, Co-cardinality, Continuity and a new Independence Property. Since this class encompasses the utilitarian rule and since there are members of this family which almost coincide with the rules of rank dictatorship this family bridges the gap between pure utilitarianism and rank dictatorship.

Keywords: social choice, utilitarian rule, rank dictatorship, welfare orderings.

1. INTRODUCTION

This paper is concerned with a class of social welfare orderings which have characteristics of Rawls’ difference principle and Bentham’s utilitarian rule at the same time. The orderings considered can be represented by social evaluation functions which are weighted sums of the components of the well ordered utility vector \((u_1, \ldots, u_n)\) (if \(i < j\) then \(u_i \leq u_j\). Thus the weights do not depend on the number of individual \(i\), but on the rank of his or her utility level. This family of welfare orderings is characterized by the Strong Pareto Property, Co-cardinality, Continuity, and an Independence Property. This last property has not been dealt with in the literature. It implies that the social evaluation functions are separable in the components of the ordered utility vector. It is weaker than the known axiom which allows the elimination of the influence of indifferent individuals (cf. e.g. d’Aspremont and Gevers (1977), Deschamps and Gevers (1978), Roberts (1980)). Since these welfare orderings encompass the utilitarian rule and since there are members of this family which almost coincide with the rules of rank dictatorship this family bridges the gap between pure utilitarianism and rank dictatorship. Unlike these orderings being of extreme nature our family in general takes into account utility levels and utility gains. Therefore the welfare orderings proposed in the

following are well suited for making judgements on the distribution of welfare (cf. Sen (1974)).

Section 2 defines the notation. In Section 3 the new property of separability with respect to ordered utility vectors is proposed and discussed. The welfare orderings described above are characterized. It is shown that one cannot dispense with one of the properties used. Section 4 concludes the paper.

2. NOTATION

We consider a society consisting of \( n \geq 3 \) individuals. Let \( \mathbb{R}^n \) be the \( n \)-dimensional Euclidean space. We make the assumption of welfarism (cf. Sen (1977)). Thus social welfare depends only on the individuals' utilities which are described by vectors \( u = (u_1, \ldots, u_n) \in \mathbb{R}^n \) where \( u_i \) denotes individual \( i \)'s utility level (\( 1 \leq i \leq n \)).

A social welfare ordering \( R \) is a complete, transitive, and reflexive binary relation on \( \mathbb{R}^n \). \( uRv \) means that \( u \) is at least as good as \( v \) (\( u, v \in \mathbb{R}^n \)); \( P \) and \( I \) define the strict preference and indifference relation corresponding to \( R \).

In the following individual positions play an important role. For every \( u \in \mathbb{R}^n \) we define the vector \( u_{[1]} = (u_{[1]}, \ldots, u_{[n]}) \). \( u_{[1]} \) is generated by a permutation of the components of \( u \), and its components are ascendingly ordered \( u_{[i-1]} < u_{[i+1]} \) for \( 1 < i < n - 1 \). Considering the ordered vector \( u_{[1]} \) one forgets all about the individuals' identity and looks only at the utility levels, without taking into account to whom they belong.

3. A CHARACTERIZATION THEOREM

In this paper we shall characterize the set of welfare orderings \( R \) which are represented by a social evaluation function of the form

\[
W(u) = \sum_{i=1}^{n} \gamma_i u_{[i]}, \quad \gamma_i > 0, \quad \sum_{i=1}^{n} \gamma_i = 1
\]

(\(^*\))

This class comprises pure\(^1\) utilitarianism (take \( \gamma_i = 1/n, \ i = 1, \ldots, n \)) exactly and Rawls' difference principle (choose \( \gamma_1 \approx 1 \), and \( \gamma_i \approx 0, \ i = 2, \ldots, n \)) approximately.\(^2\)