A NOTE ON THE EXISTENCE OF ALFVÉN SURFACE WAVES

(Research Note)

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Abstract. The Alfvén surface waves can arise due to the discontinuity in the Alfvén speed across the interface along which these waves propagate. This note studies the relationship between \(v_{A1}\) and \(v_{A2}\) which is required for the existence of Alfvén surface waves in low-\(\beta\) plasma.

Recently, the study of Alfvén surface waves has acquired a new interest as the resonant absorption of these waves provides a mechanism for irreversible heating of laboratory plasma (Uberoï, 1964, 1972, 1974; Hasegawa and Chen, 1974, 1976; Grossman and Tataronis, 1973; Uberoi and Somasundaram, 1980) and Coronal plasma (Ionson, 1978; Wentzel, 1979; Roberts, 1980).

The Alfvén surface waves can arise due to the discontinuity in the Alfvén speed across the interface, say \(x = 0\), along which these waves propagate. In a realistic situation the Alfvén speed will change from \(v_{A1}\) say to \(v_{A2}\) across a small layer 'a'. The surface waves irreversibly heat the plasma by coupling its energy into the heavily damped kinetic Alfvén waves within this layer, the coupling arising near the critical point \(x = x_c\) at which the phase velocity \(v_{ph}\) of the surface wave meets the Alfvén velocity \(v_A(x)\) i.e., \(v_{phc} = v_A(x_c)\).

Since, the Alfvén heating mechanism depends on the surface wave resonance it is important to understand the relationship between \(v_{A1}\) and \(v_{A2}\) which is required to excite a surface wave with a particular propagation direction with respect to the magnetic field direction and a given phase speed. In this note I discuss this relationship for low-\(\beta\) plasmas which are more interesting astrophysically.

Without going into details of deriving the dispersion relation of Alfvén surface waves along an interface \(x = 0\) formed by two compressible conducting media, referring the readers to details of this derivation in Wentzel (1979) and Roberts (1980) work, we write down the dispersion relation as

\[
\rho_1(k^2v_{A1}^2 - \omega^2)(m_1^2 + l^2)^{1/2} + \rho_2(k^2v_{A2}^2 - \omega^2)(m_2^2 + l^2)^{1/2} = 0, \tag{1}
\]

where \(\rho_1\) and \(\rho_2\) are equilibrium densities in medium 1 and 2, \(v_{A1}\) and \(v_{A2}\) are Alfvén speeds given as \(v_{A1,2} = |B_{1,2}|/(\mu_0\rho_{1,2})^{1/2}\), with \(B\) being the external magnetic field directed along \(x\) direction, and

\[
m_{1,2}^2 = \frac{(k^2c_{1,2}^2 - \omega^2)(k^2v_{A1,2}^2 - \omega^2)}{k^2c_{1,2}^2v_{A1,2}^2 - (c_{1,2}^2 + v_{A1,2}^2)\omega^2}, \tag{2}
\]

where \(c_{1,2} = (\gamma\rho_{1,2}/\rho_{1,2})^{1/2}\) is the sound speed.

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For the surface wave solutions to be evanecents in both media \((m_1^2 + l^2)^{1/2}\) and \((m_2^2 + l^2)^{1/2}\) both should be positive.

We consider the low-\(\beta\) plasma which is astrophysically the most interesting case. Taking the Alfvén speed to greatly exceed the sound speeds on both sides of the interface, we get from (2)

\[
m_1^2 \approx k^2 - \frac{\omega^2}{v_{A1}^2}, \quad m_2^2 \approx k^2 - \frac{\omega^2}{v_{A2}^2}
\]

and thus reduce Equation (1) to

\[
\rho_1 (k^2 v_{A1}^2 - \omega^2) \left( l^2 + k^2 - \frac{\omega^2}{v_{A2}^2} \right)^{1/2} + \rho_2 (k^2 v_{A2}^2 - \omega^2) \left( l^2 + k^2 - \frac{\omega^2}{v_{A1}^2} \right)^{1/2} = 0. \tag{3}
\]

The pressure balance condition

\[
p_1 + \frac{B_1^2}{2\mu} = p_2 + \frac{B_2^2}{2\mu}
\]

for low-\(\beta\) plasma implies \(\rho_1 v_{A1}^2 \approx \rho_2 v_{A2}^2\). Further taking \(l = K \sin \theta\) and \(k = K \cos \theta\) and \(\omega/k = \nu_{ph}\), Equation (3) reduces to

\[
v_{A2} (v_{A1}^2 - \nu_{ph}^2) (v_{A2}^2 - \nu_{ph}^2 \cos^2 \theta)^{1/2} + v_{A1} (v_{A2}^2 - \nu_{ph}^2) (v_{A1}^2 - \nu_{ph}^2 \cos^2 \theta)^{1/2} = 0. \tag{4}
\]

From (3) we note that for surface waves to exist \((k^2 v_{A1}^2 - \omega^2)\) and \((k^2 v_{A2}^2 - \omega^2)\) must have opposite signs, so one of the Alfvén speed must be above the critical speed \(\nu_{ph}\) while the other should be below it. Furthermore, both the speeds should be greater than \(\nu_{ph} \cos \theta\), in order that the solution should be evanescent. If \(v_{A2}\) is taken to be larger of the two Alfvén speeds \(v_{A1}\) and \(v_{A2}\), \(v_{A2} \geq v_{ph}\) and \(v_{ph} \cos \theta \leq v_{A1} < v_{ph}\). Thus the relation between \(v_{A1}\) and \(v_{A2}\) required for the existence of the Alfvén surface wave as obtained from Equation (4) is

\[
\frac{v_{A2}^2}{\nu_{ph}^2} = 1 + \frac{(\nu_{ph}^2 - v_{A1}^2) \sin^2 \theta}{v_{A1}^2 (1 + \sin^2 \theta) - \nu_{ph}^2}. 
\]

Curves of \(v_{A2}^2\) versus \(v_{A1}^2\) are shown in Figure 1 for several values of \(\theta\).

As pointed out by Wentzel (1979) the astrophysically most interesting hydromagnetic surface wave is the Alfvénic surface wave, which occurs when \(p \ll B^2/2\mu\) on each side of the interface and \(k/l \ll 1\). As a numerical example, therefore consider \(\theta = 80^\circ\), which makes \(k \ll l\). The Alfvén velocities involved in the solar coronal conditions are of the