ON THE RELATION BETWEEN THE SOLAR ACTIVITY CYCLE AND THE SOLAR TIDAL FORCE INDUCED BY THE PLANETS

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Abstract. The cross-correlation coefficient \( \gamma(t) \) of the solar tidal force induced by the planets \( f(x + t) \) with the sunspot number \( g(x) \) during a period of 44 years is about \(-0.7\) when \( t \) is about \(-2\) years. This fact will be useful for predicting solar activity. The solar tidal force was calculated from 1928 to 1971 for every degree on the equatorial plane and every time every planet moves one degree. As the solar tidal force, we used the moving annual average by months of the square of the vertical tidal force on the sun, and as the sunspot number we used the Zürich mean annual sunspot number.

1. Introduction

Prediction of the solar activity is necessary not only for terrestrial affairs but also for artificial satellites, as the solar proton-rays, X-rays, ultraviolet and radio waves vary greatly, depending on the solar activity, but the reason for variation in solar activity is not yet clear. In order to find the method of prediction of solar activity with full knowledge of its mechanism, calculation was made of the correlation of the solar tidal force induced by the planets with the sunspot number.

2. Cycle of the Solar Tidal Force

The tidal force on the particles in the solar atmosphere is approximately expressed as

\[
Fr = GmM \frac{r}{R^3} (3 \cos^2 \theta - 1) \quad (1)
\]

\[
Ft = -\frac{3}{2}GmM \frac{r}{R^3} \sin 2\theta \quad (2)
\]

where \( Fr \) = vertical tidal force at \( r \) from the solar center, \( Ft \) = horizontal tidal force at \( r \) from the solar center, \( G \) = gravitational constant, \( m \) = mass of particles at \( r \) from the solar center, \( M \) = mass of planets, \( R \) = distance from the solar center to the planet, \( r \) = distance from the solar center to the particles in question, and \( \theta \) = angle between \( R \) and \( r \).

Table I shows the values of masses, eccentricity, mean distances from the sun, periods, and the maximum, mean and minimum tidal forces induced by the planets, relative to the values of the earth. The tidal forces induced by Jupiter, Venus, earth and Mercury are relatively strong, and the forces induced by Neptune and Pluto are negligible. The force by Mercury is nearly equal to that by the earth, but its orbital inclination is large (\( i = 7.0 \) degrees) and the period is less than one-fourth of that of
TABLE I
Planetary orbital elements and tidal forces on the sun induced by the planets (Epoch 1964, 1, 6.0).

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mass</th>
<th>Eccentricity</th>
<th>Mean distance</th>
<th>Sidereal period</th>
<th>Tidal force Maximum</th>
<th>Mean</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.0556</td>
<td>0.2056</td>
<td>0.3871</td>
<td>0.2408</td>
<td>1.912</td>
<td>0.959</td>
<td>0.547</td>
</tr>
<tr>
<td>Venus</td>
<td>0.8172</td>
<td>0.0068</td>
<td>0.7233</td>
<td>0.6152</td>
<td>2.204</td>
<td>2.160</td>
<td>2.116</td>
</tr>
<tr>
<td>Earth</td>
<td>1.0000</td>
<td>0.0167</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.052</td>
<td>1.000</td>
<td>0.951</td>
</tr>
<tr>
<td>Mars</td>
<td>0.108</td>
<td>0.0934</td>
<td>1.5237</td>
<td>1.8808</td>
<td>0.041</td>
<td>0.031</td>
<td>0.023</td>
</tr>
<tr>
<td>Jupiter</td>
<td>318.4</td>
<td>0.0482</td>
<td>5.2024</td>
<td>11.866</td>
<td>2.623</td>
<td>2.261</td>
<td>1.963</td>
</tr>
<tr>
<td>Saturn</td>
<td>95.22</td>
<td>0.0537</td>
<td>9.5728</td>
<td>29.618</td>
<td>0.128</td>
<td>0.109</td>
<td>0.093</td>
</tr>
<tr>
<td>Uranus</td>
<td>14.58</td>
<td>0.0445</td>
<td>19.135</td>
<td>83.702</td>
<td>0.0024</td>
<td>0.0021</td>
<td>0.0018</td>
</tr>
<tr>
<td>Neptune</td>
<td>17.264</td>
<td>0.0119</td>
<td>29.968</td>
<td>164.05</td>
<td>0.00067</td>
<td>0.00064</td>
<td>0.00062</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.92</td>
<td>0.2455</td>
<td>39.265</td>
<td>246.04</td>
<td>0.000035</td>
<td>0.000015</td>
<td>0.0000079</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.963</td>
<td>6.523</td>
<td>5.695</td>
</tr>
</tbody>
</table>

the earth, so the effect of acceleration by the force may be small. First, consider only Jupiter, Venus and the earth. As both the tidal force and period of Jupiter are the largest, we consider the period relative to Jupiter. When the angular velocity of Jupiter is denoted by \( \omega_j \) and others by \( \omega \) and the cycle of Jupiter is denoted by \( v_j \) and others by \( v \), the period relative to Jupiter, \( T \), is expressed as

\[
\omega_j T + 2\pi = \omega T
\]

or

\[
T = \frac{2\pi}{|\omega - \omega_j|} = \frac{1}{|v - v_j|}.
\]

From the above formulae, the period of Venus relative to Jupiter is 0.64884 year = 236.9 days. Since the tidal forces become the same when the relative phases of the two planets are the same and opposite, the period of tidal forces induced by Jupiter and Venus is 0.3244 year = 118.5 days = 2 \times 59.2 days, very close to twice the 60-day sunspot cycle. From Equation (3) we can also obtain the period of the earth relative to Jupiter, 2 \times 0.5460 years. If the integral multiple of the tidal period of Venus relative to Jupiter is equal to the integral multiple of the tidal period of the earth relative to Jupiter, the least common multiple which is close to the integral multiple of Jupiter's period must be the period of the tidal forces on the sun induced by Jupiter, Venus and the earth. The period is

\[
0.54601m = 0.32442n \quad \text{(years)}
\]

where \( m \) and \( n \) are integers.

If there are proper \( m \) and \( n \) satisfying the above condition, the solar tidal force might have a definite period.

If \( m = 41 \) and \( n = 69 \), the left-hand side of the above equation = 22.386 years, and the right-hand side of the above equation = 22.385 years.

The difference between the left-hand side and the right-hand side of the above