THE ENERGY OF ELECTRIC CURRENT SHEETS

I. Models with Moving Magnetic Dipoles

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Abstract. This paper treats two problems on the formation of electric current sheets in the highly electrically conducting solar atmosphere. The first problem concerns a vertical current sheet formed by decreasing the distance between a pair of parallel magnetic line-dipoles lying on the photosphere. The solution to this problem was given previously by Priest and Raadu. With an interest in the flare phenomenon, they derived a formula for the energy stored through the presence of the current sheet. We show that this formula is incorrect. Firstly, there is an error of sign in the derivation of Priest and Raadu, so that, when corrected, the formula gives a negative value for the stored energy. Secondly, the formula is shown to refer to an energy quite different from the free energy associated with the current sheet. To calculate for the current free energy, it is important to account for the 'frozen-in' condition in the highly conducting photosphere.

The second problem of the paper concerns the current sheet formed by increasing the distance between the pair of line-dipoles. A different field configuration results, with a curved current sheet lying transverse to the vertical. An analysis of the energy properties is given, to compare with the properties of the Priest−Raadu model.

1. Introduction

Magnetic fields with neutral points are liable to form electric current sheets in a highly conducting medium when the sources of the fields are moved. This process, taking place in the solar atmosphere, has been regarded to be an important means of storing energy for release in a flare (Dungey, 1958; Sweet, 1969; Parker, 1963; Syrovatskii, 1969). Priest and Raadu (1974, 1975) constructed an elegant two-dimensional model to simulate the formation of a current sheet when two pairs of bipolar sunspots move towards each other on the photosphere. As an idealisation, the two pairs of sunspots are represented by line-dipole magnets, with their axes aligned along the x-axis. The initial field configuration is sketched on the left in Figure 1 and the final field configuration, bearing the electric current sheet is shown in Figure 2. Priest and Raadu derived a formula for the energy stored in the current sheet. We will show in Section 2 that the formula of Priest and Raadu is incorrect. The derivation of Priest and Raadu actually contains a mistake. When that mistake is corrected, their formula yields negative values for the stored energy. This apparently paradoxical result can be resolved with the realisation that the Priest−Raadu formula is for an energy quite distinct from the energy stored through the presence of the current sheet. The matter will be clarified with a return to the basic principles of electromagnetism. The physical steps for calculating the true free energy of the current sheet will also be given. It will be found that these steps lead
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Fig. 1. The potential states of two dipoles at separations $2a_0$ and $2a$.

Fig. 2. Field configuration of a decreased dipole separation with a vertical current sheet.

to a positive energy as to be expected from the physical idea of energy storage. In Section 3, we go on to consider the current sheet formed by moving the pairs of sunspots apart rather than towards each other. An interesting result to be found is that the solution is radically different from the Priest–Raadu solution in having a curved transverse current sheet instead of a vertical one. This situation has not been treated previously.* We will present the solution to discuss its energy properties in comparison with those of the Priest–Raadu solution. We conclude with a discussion in Section 4.

2. The Priest–Raadu Model

Priest and Raadu (1975) considered the initial magnetic field due to two parallel line-dipoles located at $x = \pm a_0$, as shown on the left in Figure 1. Using complex

* After this paper was submitted for publication, we were informed by E. Priest, that the problem of the transverse current sheet has been attempted by T. J. Tur in his unpublished thesis (1976).