STATISTICAL MECHANICS OF VELOCITY AND MAGNETIC FIELDS IN SOLAR ACTIVE REGIONS

V. KRISHAN

Indian Institute of Astrophysics, Bangalore 560 034, India

(Received 19 December, 1983; in final form 29 October, 1984)

Abstract. A statistical mechanics of the velocity and magnetic fields is formulated for an active region plasma. The plasma subjected to the conservation laws emerges in a most probable state which is described by an equilibrium distribution function containing a lagrange multiplier for every invariant of the system. The lagrange multipliers are determined by demanding that the measured expectation values of the invariants be reproduced. For a numerical exercise, we have assumed some probable values for these invariants. The total energy of a coronal loop is estimated from energy balance considerations. Doppler widths of the UV and EUV lines excited in the coronal loop plasma give a measure of the root-mean-square velocities. Measurements of magnetic helicity are not available for the solar corona.

1. Introduction

Solar active regions are believed to be dominated by loop like or arch like structures in emission. The spatial structure of these loops outlines the magnetic field geometry, which may be current free or force free, Vaiana and Rosner (1978). Sakurai (1976) has studied the motion of prominences of the arch and the loop type, deriving the equations of nonlinear evolution of MHD plasma system making use of the principle of least action. The time development of the prominence plasma exhibits various phases of motion. It is the phase showing turbulent motions without any rising motion, that leads to the steady loop system. In earlier papers (Krishan, 1983a, b) a steady-state model of active region coronal loops was presented. The active region plasma is treated as a turbulent magnetofluid. This magnetofluid when subjected to the invariance of total energy, the magnetic helicity and the toroidal and poloidal magnetic fluxes acquires a temperature profile which agrees well with the observed temperature structure of the cool core and hot sheath type of loops. The spatial widths of the UV and EUV lines excited in these loops were calculated and were found to be following the observed gradation (Krishan, 1983b). The statistical theory of incompressible magnetohydrodynamic turbulence as described by Montgomery et al. (1978) was used in order to delineate the spatial configuration of active region coronal loops. The main features of the theory consist of using the MHD equations for an incompressible fluid. The magnetic and velocity fields are expanded in terms of Chandrasekhar–Kendall functions. A single Chandrasekhar–Kendall function represents a force-free state, the superposition does not. The pressure profile of the plasma is obtained from a poisson equation for the mechanical pressure as a function of the velocity and magnetic fields. Taylor (1974, 1975, 1976) conjectured that the decay of energy to a minimum value compatible with a conserved value of magnetic helicity leads to a force-free state.
representable by a single Chandrasekhar–Kendall function. In this state of minimum energy, one can simultaneously invoke the constancy of total energy and magnetic helicity. Montgomery et al. (1978) introduced the toroidal and the poloidal magnetic fluxes as additional invariants. This resulted in several states being accessible for a fixed value of the ratio of toroidal and poloidal magnetic fluxes and for a fixed value of the axial and azimuthal mode numbers \((n, m)\), respectively. In the present paper, we present this steady state as an equilibrium ensemble. The statistical mechanics of the velocity and magnetic fields is formulated in a phase space whose coordinates are the real and imaginary parts of the expansion coefficients. The success of the lowest mode state \((m = n = 0)\) in accounting for the temperature profile of the cool core and hot sheath loops has provided the motivation for studying the statistical distribution of the velocity and the magnetic fields in this particular state \((m = n = 0)\). An equilibrium distribution is assumed in which the Lagrange multipliers are determined by requiring the expectation values of the energy, the magnetic flux and the magnetic helicity to match the observed average values of these conserved quantities. An estimate of the total energy in the coronal loop can be made using the energy balance arguments Levine and Withbroe (1977). There are no direct measurements of the magnetic helicity and magnetic fluxes in the coronal loops. Therefore, the values of these quantities assumed here to be indicative of the actual values could serve as a prediction to be verified by possible future observations. It may be appropriate to point out that the measurement of the invariants of MHD turbulence in the solar wind has been achieved Matthaeus and Goldstein (1982). In the next section, the canonical distributions for the expansion coefficients of the fields and for the conserved quantities are given. The statistical distribution of velocity fields has been derived for the prominence plasma Jensen (1982). The present work, in addition describes the distributions of magnetic helicity and magnetic fluxes. Assuming a Gibbs distribution for the system enables us to determine the magnitude as well as the probability distribution of fluctuations in the velocity and magnetic fields. The role of these fluctuations in producing large-scale coherent structures is one of the most important revelations of the MHD turbulence theory. The study of correlations between fluctuations give us clues about the kinds of MHD modes like Alfvén waves excited in the plasma. According to one suggestion, the heating and acceleration of plasma particles in a coronal loop is achieved through Alfvén waves propagating in the opposite legs of the loop. Such a situation corresponds to a finite velocity fluctuation associated with a zero magnetic field fluctuation as discussed by Matthaeus and Goldstein (1982). The presence of coherent loop like structures in the solar corona provides an appropriate system for the applications of the results of MHD turbulence theory.

2. The Equilibrium Distribution Function

The equations describing an incompressible ideal MHD turbulent plasma are:

\[
\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = \left( \nabla \times \mathbf{B} \right) \times \mathbf{B} - \nabla P ,
\]