ON THE DIFFERENCE IN DARKNESS BETWEEN SUNSPOTS

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Abstract. The effects of the magnetic field as well as the velocity field on sunspot equilibrium are discussed. The gas pressure difference, $\Delta P$, between a spot and the environments in the same horizontal layer is primarily determined by the magnetic field. Using recent model atmospheres we find that $\Delta P$ shows a maximum value, $\Delta P_{\text{max}}$, at a depth of $650 \pm 150$ km below the photosphere. The value of $\Delta P_{\text{max}}$ suggests that the curvature of the field lines is important for the equilibrium.

It appears that, at an optical depth of unity in the umbra, the density has a value close to that of the environment at the same geometric depth (see Figure 4). If such is the case the expression for the umbra temperature (Equation (15)) may be considerably simplified (Equations (17) and (18)) and compared with observations.

1. Introduction

It was pointed out by Alfvén (1943) that a static equilibrium in a vertical magnetic field is possible only if the temperature in the field is lower than in the environment. The work of Alfvén was extended by Cowling (1957) and by Dicke (1970) who showed that the darkening of a disturbed region on the solar surface was determined completely by the velocity and the magnetic field distributions in the observable layers.

Using inductive methods, magnetohydrostatic sunspot models for the deeper layers have been studied by Schlüter and Temesváry (1958), Chitre (1963), Deinzer (1965), Yun (1968, 1970, 1971) and Meyer et al. (1974). Based on empirical sunspot models the levels around an optical depth unity have been discussed by Jakimiec (1965), Zwaan (1965) and Mattig (1969). It is interesting to compare the pressure difference between the umbra and the surroundings in the same horizontal layer as derived by the different authors; Deinzer (1965) finds that the maximum pressure difference occurs at about 3000 km below the photosphere, whereas the corresponding value given by Mattig (1969) is approximately 800 km. Thus, the two methods appear to give different results.

During recent years the accuracy in the empirically determined sunspot models has improved considerably. Comparing the models of Kjeldseth Moe and Maltby (1974), Zwaan (1974, 1975), and Stellmacher and Wiehr (1975) we find differences comparable to those found between different photospheric models. Thus, reliable boundary conditions for the top of the umbra are available. The aim of this paper is to study sunspot equilibrium using these empirically determined sunspot models.

2. Gas Pressure and Magnetic Field

For a time-independent velocity field we may write

$$\rho v \cdot \nabla v = -\nabla P + \rho g + j \times B,$$

(1)
where $\rho$ is the density, $\mathbf{v}$ the velocity, $P$ the gas pressure, $g$ the gravitational acceleration and $\mathbf{B}$ the magnetic induction. Neglecting the displacement current, the current density is

$$\mathbf{j} = \frac{1}{\mu} \nabla \times \mathbf{B},$$

(2)

where $\mu$ is the permeability. Observations of the velocity field (Maltby, 1964) indicate that the azimuthal velocity component is only important for irregular spots and sunspot groups. Thus, we will assume $v_\phi = 0$. Although Piddington (1976) has suggested that a twisted magnetic field may be a necessary condition for stability in sunspots, Meyer et al. (1977) have questioned this conclusion. Yun (1971) carried through the calculations with $B_\phi \neq 0$; as his results are only slightly different from the results for $B_\phi = 0$, we will assume axial symmetry and neglect the azimuthal component of the magnetic field in the following. Subject to these assumptions Equations (1) and (2) give, in cylindrical coordinates:

$$\frac{\partial P}{\partial r} = \frac{B_z}{\mu} \left( -\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) - \rho \left( v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} \right),$$

(3a)

$$\frac{\partial P}{\partial z} = -\rho g - \frac{B_z}{\mu} \left( -\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) - \rho \left( v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right).$$

(3b)

Integration of Equation (3a) from the spot centre ($r = 0$) to the photosphere ($r = a$) gives the pressure difference,

$$\Delta P = \frac{1}{2\mu} B_z^2 (r = 0) + \frac{1}{\mu} \int_0^a B_z \frac{\partial B_r}{\partial z} \, dr - \int_0^a \frac{1}{2\rho} \left( \frac{\partial v_r^2}{\partial r} + 2v_z \frac{\partial v_z}{\partial z} \right) \, dr,$$

(4a)

where

$$\Delta P = P(r = a) - P(r = 0) = P_0 - P(r = 0).$$

(4b)

As observations refer to positions very close to $r = 0$ we have not followed Dicke (1970) who splits the integration in two parts, the first integration extending over the central parts of the umbra.

Regarding the layers around optical depth unity we know from the observations of weak spectral lines that the Evershed flow is concentrated in channels (Beckers, 1969). Thus, the contribution from the last term in Equation (4a) should be weighted over the azimuthal angle. In order to satisfy the continuity equation it is likely that $v_z > 0$. Recent observations by Bønes (1975) of the wings of the Mg $b_1$ line have shown that $v_r$ probably decreases quickly with height. As $\partial v_r/\partial r > 0$, $v_z > 0$ and $\partial v_z/\partial z < 0$, the two terms $\partial v_r^2/\partial r$ and $2v_z \partial v_z/\partial z$ will have opposite signs and probably be of comparable magnitude (approximately equal for $v_r(r = a) = 5$ km s$^{-1}$, $a = 10^4$ km, $v_z = 0.1$ km s$^{-1}$ and $\partial v_r/\partial z = -v_z/(400 \text{ km})$). This suggests that the contribution to $\Delta P$ from the Evershed flow is considerably less than the contribution from the magnetic field.