MAGNETOACOUSTIC SURFACE GRAVITY WAVES

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Abstract. The effect of gravity on the surface magnetoacoustic waves may be important when considering applications to solar and laboratory plasmas. The linear magnetoacoustic waves, which may appear in a configuration with an interface between two plasmas or a plasma and an ordinary gas, are studied. Compressibility and gravity are taken into account. The different types of couplings between internal and surface modes are analyzed. Magnetoacoustic surface waves are studied in detail in a configuration consisting of an interface between an isothermal plasma and an ordinary gas. The possible regions where these modes may exist are discussed. A general way of grouping and classifying the complicated spectra of modes is presented. New groups of modes appear as a consequence of gravity and stratification, in addition to those already present in the absence of gravity. The results may be of help in studying more complicated cases.

1. Introduction

The study of the oscillations and waves in the Sun has aroused great interest due to its relation with helioseismology, the structure of the Sun and related matters (Campbell and Roberts, 1989; for recent reviews see Brown, Mihalas, and Rhodes, 1986; Thomas, 1983; Hughes and Proctor, 1988; Deubner and Gough, 1989; Bahcall and Ulrich, 1988). In particular, magnetoacoustic surface waves are of interest in connection with several phenomena that occur in the atmosphere of the Sun (see, for example, Miles and Roberts, 1989, in which the relevance of surface waves in the Sun is discussed and additional references are given), in astrophysics (Aly, 1986), and in laboratory experiments involving interfaces of different types. Some of these phenomena are the running penumbral waves (see, for instance, Nye and Thomas, 1974, 1976; Small and Roberts, 1984), coronal heating (Hollweg, 1987a, b), etc. As a first approximation to these configurations, gravity has been neglected in many of the works (see, for instance, Roberts, 1981a, b; Abdelatif and Thomas, 1989; Miles and Roberts, 1989). Nevertheless, it is expected that gravity (or plasma acceleration in laboratory experiments) may play an important role in these phenomena. If gravity and compressibility are both taken into account, considerable mathematical complications arise owing to the stratification of the plasma. As a consequence, these problems can in general be solved only by means of numerical methods, if realistic density, magnetic field, and temperature profiles are assumed. It is a common feature in problems of this sort that the spectrum of surface oscillations exhibits several discontinuous branches, as well as forbidden wave number bands, for which no surface mode exists.

This property of the surface gravity modes can be observed even in very simplified

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situations as in incompressible viscous cases (e.g., Chandrasekhar, 1961; Gratton and González, 1986) or in non viscous compressible plasmas (González and Gratton, 1990; Miles and Roberts, 1989). Physically the forbidden bands arise from wave number \( k \) intervals for which the corresponding penetration depth is such that it can not satisfy the boundary conditions for surface (i.e., localized near the interface) modes.

The aim of this work is to study the properties of the spectrum of linear modes and the origin of the forbidden bands by analyzing a simple configuration. The proposed model may then be used as a guide when considering more complicated situations by numerical methods.

### 2. The Model

We shall consider ideal MHD. The basic equations will then be

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 ,
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \rho \mathbf{g} - \frac{1}{4\pi} \mathbf{B} \times (\nabla \times \mathbf{B}) , \tag{1}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) .
\]

In Equations (1), \( \mathbf{g} = -g \mathbf{e}_y \) denotes gravity, \( \mathbf{B} \) the magnetic field, \( \rho \) the density, \( \mathbf{v} \) the velocity, and \( p \) the pressure. In the unperturbed state, there is an interface at \( y = 0 \) across which \( \rho, \mathbf{B}, p \) may vary discontinuously. For simplicity, in the present paper we shall consider only configurations in which the unperturbed magnetic field is everywhere parallel to the interface, and the medium is stratified in \( y = \text{const.} \) planes. No mass flow is considered. The equilibrium condition for the model can then be written as

\[
\frac{d}{dy} \left( \rho + \frac{\mathbf{B}^2}{8\pi} \right) = -\rho g . \tag{2}
\]

The linear adiabatic perturbations of this equilibrium satisfy the equation (Gratton, Gratton and González, 1988)

\[
\frac{d}{dy} \left[ H \left( \frac{1}{M} - 1 \right) \frac{d \zeta}{dy} \right] + k_y^2 \zeta \left[ H - g \frac{d\rho}{dy} - g \frac{d}{dy} \frac{H}{M \omega^2} - g^2 \frac{k_y^2 H}{M \omega^4} \right] = 0 , \tag{3}
\]

where \( \zeta \) represents the \( y \)-component of the displacement of a normal mode of the type

\[
\delta Q = q(y) \exp[i(k_y \cdot \mathbf{x} - \omega t)] , \tag{4}
\]

with \( k_y = (k_x, 0, k_z) \). The following definitions have been used in (3):

\[
H = \rho (C_A^2 k_y^2 - \omega^2) , \quad M = 1 - \frac{k_y^2}{\omega^2} (C_A^2 + C_S^2) + \frac{C_A^2 C_S^2 k_y^2 k_r^2}{\omega^4} , \tag{5}
\]