NON-LTE LINE FORMATION IN A MAGNETIC FIELD

I: Non-Coherent Scattering and True Absorption

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Abstract. The formation of a Zeeman-multiplet by noncoherent scattering and true absorption in a Milne-Eddington atmosphere is considered assuming a homogeneous magnetic field and complete depolarization of the atomic line levels. The transfer equation for the Stokes parameters is transformed into a scalar integral equation of the Wiener-Hopf type which is solved by Sobolev's method in closed form. The influence of the magnetic field on the mean scattering number in an infinite medium is discussed.

The solution of the line formation problem is obtained for a Planckian source function of the form

\[ B(T(\tau)) = B_0 (1 + A_0 \tau + \sum_{i=1}^{N} A_i \exp(-a_i \tau)). \]

This solution may be simplified by making the 'finite field approximation', which should be sufficiently accurate for practical purposes.

1. Introduction

The theory of line formation in the presence of a magnetic field was developed for LTE by Unno (1956) and Stepanov (1958a, b). This work was continued and generalized by Rachkovsky (1961, 1962), Stepanov (1962), Mattig (1964), Moe (1968), and by Staude (1969, 1971) and Beckers (1969) who took into account both anomalous dispersion and inhomogeneous magnetic fields.

Scattering of radiation in a Zeeman multiplet in the process of line formation was considered by Stepanov (1958b), Rachkovsky (1963, 1965, 1967), Obridko (1965), Katz (1968), Rees (1969), Domke (1969, 1971a, b), and Lamb and ter Haar (1970a, b). Most of the numerical work was devoted to coherent scattering. However, non-coherent scattering should be a better model for line formation in a magnetic field under solar photospheric conditions. Calculations of line formation by noncoherent scattering were carried out by Rees (1969) who took into account absorption and emission in the continuum. In an earlier paper (Domke, 1971b) numerical results were published for an isothermal atmosphere neglecting continuous absorption but including anomalous dispersion.

In the present paper we consider the formation of a Zeeman multiplet by non-
coherent scattering in the presence of continuous absorption. We assume a plane-
parallel semi-infinite atmosphere and a homogeneous magnetic field. We assume that
\( \beta = \gamma/k_L \) is constant throughout the atmosphere (Milne-Eddington atmosphere) where
\( k_L \) is a characteristic line volume absorption coefficient for the Zeeman multiplet and
\( \gamma \) the volume coefficient of continuous absorption. Further we assume that the re-
emission of the truly absorbed radiation energy (in the continuum and in the line)
takes place according to LTE. As was shown by Lamb and ter Haar (1970a), complete
atomic level depolarization should be a good approximation for scattering in solar
photospheric lines commonly used in magnetographic measurements.

With these assumptions we can transform the equation of line radiation transfer
into a scalar integral equation for a line source function. The exact solution of this
integral equation may be found in terms of a generalized \( H \)-function which is ob-
tained in closed form. The exact solution for the Stokes parameters of the line radia-
tion emergent from the atmosphere is expressed in terms of this \( H \)-function. Zero field
and finite field approximations are discussed.

2. The Transfer Equation

The polarized radiation may be described by a four component Stokes vector \( \mathbf{I}(\tau, v, \mu, \varphi) \) depending on the line optical depth \( \tau \), frequency \( v \), the angle \( \theta = \arccos \mu \) between the outer normal to the atmosphere and the direction of light propagation,
and the azimuth \( \varphi \). It was shown earlier (Domke, 1971a) that, with the assumptions
listed above, the transfer equation may be written in the form

\[
\frac{\partial}{\partial \tau} \mathbf{I}(\tau, v, \mu, \varphi) = (\alpha(v, \gamma) + \beta \mathbf{E}) \mathbf{I}(\tau, v, \mu, \varphi) - \\
- \alpha(v, \gamma) \mathbf{S}^0 S_L(\tau) - \beta \mathbf{S}^0 B(T(\tau)).
\]

(1)

Here \( S_L(\tau) \) is the scalar line source function defined by

\[
S_L(\tau) = \int_0^\infty d\nu \int_0^{2\pi} d\varphi \int_{-1}^{1} d\mu S^0 \alpha(v, \gamma) \mathbf{I}(\tau, v, \mu, \varphi) + (1 - \lambda) B(T(\tau)),
\]

(2)

\[
c = \left[ \int_0^\infty d\nu \int_0^{2\pi} d\varphi \int_{-1}^{1} d\mu S^0 \alpha(v, \gamma) S^0 \right]^{-1}.
\]

(3)

\( \alpha(v, \gamma) \) is the absorption matrix for a Zeeman multiplet which depends on \( v \) and the
angle \( \gamma \) between the magnetic field and the direction of light propagation. \( \mathbf{E} \) is the
unit matrix and \( \mathbf{S}^0 \), the Stokes vector which describes unpolarized light of the intensity
1. The single scattering albedo \( \lambda \) is assumed constant throughout the atmosphere.
\( B(T) \) is the Planck function depending on the local temperature \( T \). The action of