TRAPPED GRAVITY WAVES AND THE FIVE-MINUTE OSCILLATIONS OF THE SOLAR ATMOSPHERE

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Abstract. The various modes of hydrodynamic waves are considered for a model of the solar atmosphere which is based on the Bilderberg model and includes the effects of ionization. The atmosphere forms a 'potential well' for internal gravity waves, since the stability is low at the base (near the convection) and low again in the region of partial ionization in the chromosphere. Calculations show that there are two resonant (trapped) modes of internal gravity waves for horizontal wavelengths based on the scale of the granulation. The properties of these modes are in close agreement with the two modes of oscillation observed by Frazier (1968). Trapped acoustic modes are found to have periods too short to account for the observations.

1. Introduction

Of the many theoretical studies of the vertical velocity oscillations in the solar atmosphere, none has taken into account the effect of the partial hydrogen ionization in the low chromosphere. The purpose of this paper is to show that the partial ionization has an important effect on the mechanical properties of the atmosphere. We take as our basic working model the Bilderberg Continuum Atmosphere (Gingerich and de Jager, 1968). As we shall see, the partial ionization creates a 'potential well' for internal gravity waves in the photosphere – low chromosphere region. The resulting trapped (resonant) modes of internal gravity waves have properties in close agreement with observations of the oscillations.

Uchida (1965, 1967) has investigated the trapping of internal gravity waves in the temperature trough in the solar atmosphere. In the present work, we find that it is the ionization, and not the variation in temperature, which leads to the trapping of gravity waves. Kahn (1961, 1962) has suggested that the oscillations are due to trapped acoustic waves. In our model, it turns out that the trapped modes of acoustic waves have periods too short to account for the oscillations.

The linearized equations for waves in a general atmosphere (derived in Section 2) may be written in terms of two parameters, the Brunt-Väisälä frequency \( N \) and the sound speed \( c \). The distribution of \( N \) and \( c \) are computed for the Bilderberg model (Section 3). We then use a piecewise constant model of these distributions (described in Section 4) to calculate the trapped gravity wave modes (Section 5).

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2. Basic Equations

In this section we derive the linearized equations for waves in a general, stratified, compressible atmosphere. Let the basic unperturbed atmosphere have pressure $p_0(z)$, density $\rho_0(z)$, temperature $T_0(z)$, and specific entropy $s_0(z)$. The acceleration of gravity $g$ (in the $-z$-direction) is assumed constant, and the pressure and density are related by the equation of hydrostatic balance,

$$\frac{dp_0}{dz} = -\rho_0 g. \tag{1}$$

The parameters of importance in the final equation are the Brunt-Väisälä frequency $N$ and the sound speed $c$, defined by

$$N^2 = \frac{g}{c_p} \beta T_0 \frac{ds_0}{dz}, \tag{2}$$

$$c^2 = \left( \frac{\partial p_0}{\partial \rho_0 / s} \right), \tag{3}$$

where $c_p$ is the specific heat at constant pressure and

$$\beta = -\frac{1}{\rho_0} \left( \frac{\partial \rho_0}{\partial T_0} \right)_p$$

is the coefficient of expansion.

We introduce a small amplitude disturbance, letting the pressure, density, entropy, and velocity be, respectively, $p_0 + \rho, \rho_0 + \rho, s_0 + s,$ and $v$, where the perturbation quantities $\rho, \rho, s,$ and $v$ are small. The quantities $\rho, \rho,$ and $s$ are connected by the thermodynamic relation (combined first and second law)

$$T_0 s = \frac{c_p}{\beta \rho_0} \left[ \frac{p}{c^2} - \rho \right]. \tag{4}$$

The basic linearized equations of continuity, momentum, and energy are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_0 v) = 0, \tag{5}$$

$$\rho_0 \frac{\partial v}{\partial t} = -\nabla p + \rho g, \tag{6}$$

$$\frac{\partial s}{\partial t} + v \cdot \nabla s_0 = 0. \tag{7}$$

In Equation (7), the perturbation has been taken to be adiabatic. (This assumption is discussed further in Section 6.)

Now, let $w$ be the vertical component of the velocity $v$ and let $u$ be the horizontal component. Let the perturbation quantities have time dependence $e^{-i\omega t}$ and horizontal space dependence $e^{ik \cdot r}$ ($k = k_x i + k_y j$, $r = xi + yj$). Then, the linearized disturbance