THE NATURE OF RUNNING PENUMBRAL WAVES

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Abstract. A model of a sunspot penumbra, including the effects of magnetic field, compressibility, and buoyancy, is studied in order to identify the mode of running penumbral waves. It is found that the penumbral waves may be identified with gravity-modified magneto-acoustic waves of the 'plus' type that are vertically trapped at photospheric levels. Although most of the wave energy is contained in the penumbral photosphere and subphotosphere, the maximum vertical velocity occurs in the chromosphere where (i) the waves are evanescent and (ii) the vertical velocity is in fact observed (in Hα).

1. Introduction

Recent observations have disclosed an interesting pattern of velocity fields in sunspots. The most recent discovery is that of waves propagating radially outward in sunspot penumbrae (Zirin and Stein, 1972; Giovanelli, 1972). Zirin and Stein refer to these waves as running penumbral waves. With the further observations of Giovanelli (1974), we now have a fairly clear picture of the properties of these waves. The purpose of this theoretical paper is to study possible wave modes in a model of a sunspot penumbra in order to identify the mode of the running penumbral waves. We shall argue that the running penumbral waves are gravity-modified magneto-acoustic waves (of the 'plus' type) that are vertically trapped at photospheric levels.

Giovanelli (1974) has summarized the observations of running penumbral waves, and he presents the following picture. The waves are observed in Hα by means of their line-of-sight velocity. They occur in almost every sizable spot with a regular stable structure, but only rarely in active spots with complex structure. The waves travel outward in the penumbra at a typical speed of 15 km s^{-1}. The observed waves have periods in the range 180–240 s and horizontal wavelengths in the range 2350–3800 km. Observations near the limb have failed to reveal any horizontal motions associated with the penumbral waves, so the wave motion is predominantly vertical in Hα.

Thus far no detailed theoretical study of the mode of the running penumbral waves has appeared, although Moore (1973) has studied the related problem of the generation of penumbral waves in the umbra. Zirin and Stein (1972) tentatively identified the penumbral waves as sound waves, whereas Giovanelli (1972, 1974) identified them as Alfvén waves. The penumbral waves, with their predominant vertical motions, no doubt involve the combined effects of restoring forces due to compressibility, magnetic

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field, and buoyancy, and a complete theory should account for all three effects. This is done in the present paper.

In studying penumbral waves, we face a difficulty, in that there seems to be no complete, generally accepted penumbral model on which to base our calculations. We have therefore constructed a penumbral model for use in studying wave modes. This model (presented in Section 3), while simple enough to permit analysis of wave modes, nevertheless reproduces all of the relevant features of penumbral structure, and is in reasonable quantitative agreement with observations. We have assumed the penumbral magnetic field to be purely horizontal, but varying with height. True penumbral magnetic fields are not purely horizontal, although they may be very nearly so (Nishi and Makita, 1973). There is some disagreement over the inclination of the magnetic field in a penumbra (see Beckers and Schröter (1969) for a summary of observations). The assumption of a horizontal field here is mostly a matter of convenience; the basic mechanism we propose for the vertical trapping of penumbral waves will also work for an inclined field. We have also taken our model to be horizontally uniform – that is, we have not tried to represent the horizontal filamentary structure of a penumbra or the radial geometry.

In Section 2 we present the basic equations for waves in our model penumbral atmosphere. The basic atmosphere is completely characterized in these equations by the vertical distribution of three parameters: the sound speed c, the Alfvén velocity v_A, and the local density scale height H. In order to illustrate the properties of the various wave modes that can occur, we study the dispersion relation that holds in the case of constant c, v_A, and H. In Section 3 we present the basic penumbral model in terms of the distributions of c, v_A, and H with height. In Section 4 we show that the penumbral waves may be identified with 'plus' modes that are trapped in the photospheric-subphotospheric region in our model. We discuss these modes further in Section 5.

2. Basic Equations and Dispersion Relations

In our simplified treatment of a sunspot penumbra we shall ignore the radial spreading of magnetic field lines, and consider the undisturbed magnetic field to be purely horizontal (in the x-direction) and varying with height z; i.e., $\mathbf{B}_0=(B_0(z), 0, 0)$. We assume the field permeates an inviscid, perfectly conducting, plane stratified atmosphere with constant acceleration of gravity g $(=0.274 \text{ km s}^{-2})$ in the negative z-direction. The undisturbed pressure, density, and temperature are denoted by $p_0(z)$, $\varrho_0(z)$, and $T_0(z)$, respectively. The atmosphere is in hydrostatic equilibrium, so that

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(p_0 + \frac{B_0^2}{8\pi}\right) = -\varrho_0 g \,. \tag{1}$$

We then consider small adiabatic perturbations of this equilibrium atmosphere. We consider wave vectors only in the xz plane, and assume that the perturbation velocity $\mathbf{u}=(u, v, w)$ has the form $\mathbf{u}=\hat{\mathbf{u}} \exp i\,(k_x x-\omega t)$, with $\hat{\mathbf{u}}=\hat{\mathbf{u}}(z)=(\hat{u}(z), \hat{v}(z),$