ANALYSIS OF THE 5 MIN OSCILLATORY PHOTOSPHERIC MOTION

II: Statistical Analysis of the Oscillation as a Narrow-Band Random Process

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Abstract. A more objective statistical technique is applied directly to the four time series used in Paper I. The empirical probability density functions indicate that the measurements are realizations of a narrow-band random process with Gaussian statistics. This result allows quantitative statistical use of the mean autocorrelation function. For example, a characteristic correlation time is 23 min, and the motion becomes statistically uncorrelated over intervals greater than 40 min. The mean autocorrelation function is found to be free of secondary maxima that have been so troublesome in other analyses. The question raised in this paper is whether our statistical model of the motion as a Gaussian random process is also applicable to smaller regions on the order of 1" to 2" in size.

1. Introduction

In the first paper of this series (Cha and White, 1972) we analyzed several long (4–5 h) velocity curves produced by the oscillatory motion in weak-field regions of the solar photosphere and arrived at the following results. First, the observed time-dependence for the velocity cannot be explained in terms of the sum of a few sinusoids or the amplitude modulation of a sinusoidal carrier, instead, it appears to be a random variation with a finite correlation time. Second, the mean power spectrum for a collection of such curves is a single-peaked function (peak frequency = 3.4 mHz and bandwidth = 0.9 mHz), which lies entirely within the band between 2.0 and 5.0 mHz. Moreover, it is not a line spectrum or a combined line and continuous spectrum, which are characteristic of periodic and almost-periodic phenomena or mixed processes, respectively; but is a continuous spectrum characteristic of random processes. These properties indicate to us that the Mt. Wilson velocity curves are 'narrow-band random signals'.

However, the analysis in Paper I has not been entirely satisfactory from a statistical point of view, mainly because the procedure used to study the random amplitude and phase variations is based on a technique of dividing each time series into 'bursts'. This procedure gave an description of the average amplitude and phase variation between bursts, yet it provides almost no quantitative measures of correlation as a continuous function of time. There is no inherent property of the series that completely justifies such a segmentation, and each time series should be viewed as a continuous oscillatory variation. We now need to carry out a more objective analysis in a manner that avoids any segmentation and treats each velocity time series and its associated amplitude, phase, and frequency variation as continuous functions of time.

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In this paper, we shall attempt such an analysis by applying a formalism already developed for the statistical analysis of narrow-band random signals. In Appendix 1 we review this formalism that gives the probability distribution functions for the total signal, the amplitude, and the phase of narrow-band random processes with Gaussian statistics. The main part of this paper is devoted to the determination of probability density functions from the Mt. Wilson velocity curves. These calculations yield the statistical model of the observations given in Section 2. Section 3 is devoted to determination of the mean autocorrelation function for the Mt. Wilson data. In Section 4 we present the results of the analysis of a small number of time series observed in plages and sunspots.

2. Statistical Analysis of the Oscillatory Photospheric Motion in Weak-Field Regions: 1st Order Probability Density Functions

We shall begin our investigation of the statistical properties of the oscillatory photospheric motion by computing the 1st order probability density functions for observable features of the motion. Later, in Section 3, we shall discuss the correlation properties displayed by the motion.

2.1. Determination of Empirical Probability Density Functions

Before we can apply the ideas discussed in Appendix 1 to the four weak-field velocity curves described in Paper I, we first need to select the observables that are to be examined and the method for calculating their probability density functions.

Since the weak-field velocity curves are narrow-band signals (random or otherwise), we shall assume that the oscillatory photospheric motion is a narrow-band physical process, and hence, that we can always write

\[ v(t) = A(t) \cos[2\pi f t + \phi(t)]. \]  

where \( v(t) \) is the velocity of this motion in \( \text{km s}^{-1} \), \( A(t) \) and \( \phi(t) \) are the amplitude and phase angle, respectively, for the oscillations in \( v(t) \), and the mean frequency of the oscillations, \( f \), is about 3.3 mHz. Thus, there are four observable features we can study – \( v(t) \), \( A(t) \), \( \phi(t) \), and the instantaneous frequency, \( f_{\text{inst}} \), i.e., \( 1/2\pi \) times the derivative of the total phase, \( 2\pi f t + \phi(t) \). Further, the velocity curves, as such, form an ensemble or realizations for the first quantity, \( v(t) \). However, since we can always compute the amplitude and phase angle by constructing the analytic signal corresponding to \( v(t) \) (see Paper I for details), we also have ensembles for \( A(t) \), \( \phi(t) \) and \( f_{\text{inst}} \); and we can also calculate a density function for each of these quantities.

Note that we do not assume either that the photospheric motion is a random process or that its observables are random variables. Clearly, we wish to avoid such assumptions at the outset. On the other hand, since the results in Paper I strongly suggest that the motion is a random process that is narrow-band, we shall compare the empirical 1st order probability density functions with (1) the theoretical density functions discussed in Appendix 1; and (2) a similar set of density functions for four simulated time series.