Symbolic computation: a unified approach to studying likelihood

JAMES E. STAFFORD
Department of Statistical and Actuarial Sciences, Western Science Centre, London, Ontario, W6A 5B7, Canada

DAVID F. ANDREWS
Department of Statistics, University of Toronto, Toronto, Ontario, Canada

YONG WANG
B57 Environmental Health Centre, Health Canada, Tunney's Pasture, Ottawa, Canada K1A 0L2

We describe a set of procedures that automate many algebraic calculations common in statistical asymptotic theory. The procedures are very general and serve to unify the study of likelihood and likelihood type functions. The procedures emulate techniques one would normally carry out by hand; this strategy is emphasised throughout the paper. The purpose of the software is to provide a practical alternative to difficult manual algebraic computations. The result is a method that is quick and free of clerical error.

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Introduction

There are many likelihood and pseudo-likelihood based approaches for conducting inference in statistical theory. There exist methods based on the usual likelihood, the profile and adjusted profile likelihoods, the empirical and bootstrap likelihoods, quasi-likelihood and so on. Mukerjee and Chandra (1991), Davison et al. (1992) and McCullagh and Tibshirani (1990) investigate the relative merits of some of these methods. Lawley (1956), DiCiccio and Stern (1992), Stafford and Andrews (1993), DiCiccio et al. (1991) discuss improvements to asymptotic approximations for likelihood ratio statistics. Stafford (1992), Sprott (1973), DiCiccio (1984), and Ferguson (1992) examine asymptotic properties of score statistics and maximum likelihood estimates. All of these investigations have a common unifying theme, as they all involve a core of formal algebraic manipulations where only the likelihood function differs.

For instance, an asymptotic expansion for a maximum likelihood estimate may be obtained through the inversion of a Taylor series of the score function. Expansions for its moments and hence cumulants are calculated by finding the expected value of resulting asymptotic expansions. The Bartlett correction of a likelihood ratio statistic may be obtained by composing an expansion for a maximum likelihood estimate with a Taylor series of that statistic. The structure of all of these calculations is the same for the usual likelihood function or any of its analogues.

In this paper, we exploit these basic similarities in a set of cooperating tools that can be used to do research in likelihood or pseudo-likelihood theory. Our procedures are significantly different from algorithms that implement existing statistical theory because they may also be used to develop new statistical theory (Stafford and Andrews, 1993). The procedures are generally applicable because the calculations that we automate are quite general.

In the following demonstration of available software, the asymptotic expansion for the likelihood ratio statistic is derived and its expected value calculated, all in a general way. The statistic is then Bartlett corrected and the first moment is calculated to third order, showing the effect of the correction. As the parameter is assumed to be scalar with no nuisance parameter present the power notation discussed in McCullagh (1987) is used. The expressions derived may be found in McCullagh and Cox (1986). This demonstration uses only a few basic procedures that are available. The procedures are implemented in Mathematica Version 2.0 and the session begins after having already
literature and the expressions derived by the computer. Errors have arisen in the expressions that are found in the Ph.D. dissertation of Ferguson (1989). Occasionally differences have arisen in the expressions found in the work of Jenkins (1962), Lawley (1956), McCullagh (1987), McCullagh and Tibshirani (1990), DiCicco (1983) and Ferguson (1989). Most of these errors appear to be typographical and in the case of DiCicco (1983), they have been corrected in a subsequent publication (DiCicco, 1984). However, even typographical errors can be serious upon implementation and the error discovered in McCullagh and Tibshirani (1990) is substantial as we shall later exhibit.

The potential for a system of computer algebra tools has already been discussed in Andrews and Stafford (1993). In this paper, we explicitly describe the implementation of many of the functions introduced in this paper. In addition we describe how the tools have been extended to more important cases where there are arbitrary numbers of parameters, where inference for a parameter of interest may be conducted in the presence of a nuisance parameter and where we are interested in studying the properties of the root of a function other than the usual score function.

Symbolic computation is a powerful facility available to research statisticians. Packages like MATHEMATICA, MAPLE, MACSYMA and REDUCE are being used increasingly in areas of statistical theory where the algebra may be otherwise prohibitively complex. Stafford (1992), Stafford and Andrews (1993), Andrews and Stafford (1993) and Andrews et al. (1992) use methods of symbolic computation to derive new statistical results as well as to confirm old results. Kendall (1992) develops routines in REDUCE that aid in determining whether a particular expression is tensorially invariant. There are numerous books that describe a variety of useful applications of symbolic computation in statistics (Heller, 1991, Maeder (1991), Wagan (1991), Davenport, et al. 1988). Other contributions include Kendall (1988; 1990), Silverman and Young (1987), Young and Daniels (1990), Venables (1985) and Barndorff-Nielsen and Blaesild (1986). For a complete review of the use of symbolic computation in probability and statistics, see Kendall (1993).

The bulk of the content of this paper is in the next section, where basic functions and the underlying strategies of their implementation are discussed. The section is organized in much the same way as the typical paper where asymptotic expressions are presented. We begin with notation and proceed from the simple scalar parameter case to more complicated situations. Because a typical generic asymptotic calculation may involve, in the following order, a Taylor series or its inversion, a composition of two series expansions, the calculation of an expected value, the simplification of an expression through the use of identities and finally, the evaluation of an expression for a particular distribution, we have presented procedures automating these calculations in the same order—that is, in the order in which they would be used to perform a calculation. The description of each function is accompanied by a brief example depicting its use.