X-RAY RESONANCE SCATTERING IN A SPHERICALLY SYMMETRIC CORONAL MODEL

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Abstract. In the solar corona the opacities of some of the prominent X-ray emission lines are on the order of \( \tau \approx 1 \) over typical coronal path lengths. We present and discuss a particular solution of the radiative transfer problem involving an extended, spherically symmetric coronal shell radiating isotropic, homogeneous emission in which single-scattering also takes place. Within the context of this simplified model we find that scattered radiation is an important contribution to the total emergent resonance line flux and that for the He-like family of resonance \((r)\), intercombination \((i)\), and forbidden \((f)\) lines, the ratio \( G = (f + i)/r \) would decrease as a function of optical depth for disk-center emission in an extended spherically symmetric corona.

1. Motivation

The solar corona emits radiation that is dominated by soft X-ray emission lines. The most prominent emission lines from the high temperature, low density coronal plasma are those due to the H-like and He-like isoelectronic sequences of some of the heavier elements (C, N, O, Ne, Mg, Si, S, Ca, and Fe), and in particular the resonance lines of these ions. As first suggested by Elwert (1954, 1956) the opacities of some of these resonance lines are such that over typical coronal path lengths the emission is no longer optically thin; however neither are typical coronal optical depths very large, and the appropriate radiative transfer formulation for the modeling of coronal X-ray resonance line emission involves primarily single scattering.

The general integral equation formal solution for the transfer of resonance-line radiation under solar coronal conditions was derived by Acton (1978). In this paper we present a semi-analytical solution for the outward normal intensity in the case of an extended spherically symmetric coronal shell radiating isotropic, homogeneous emission in which single-scattering, with a Rayleigh phase function, also takes place. This highly idealized case has been addressed to investigate the complexity introduced by the relevant coronal boundary conditions in spherical geometry, and to that extent is an exploratory investigation of the feasibility of finding analytical solutions for coronal radiative transfer models. This is important because there are considerable computational advantages in basing the architecture of a numerical code on analytical solutions up to whatever point is feasible before resorting to numerical solutions. We also present this as a 'calibration solution' for numerical simulations.

2. The Observational Context

Observationally the He-like resonance lines are of more interest that the H-like lines because the resonance line of a He-like ion \((r; 1s^2 \, ^1S - 1s2p \, ^1P)\) has closely adjacent
to it in wavelength an intercombination line \((i; 1s^2 1S - 1s2p^3P)\) and a forbidden line \((f; 1s^2 1S - 1s2s^3S)\) (cf. the Grotrian diagram, Figure 5.5, in Zirin 1966) also originating from the ground state. The wavelengths of the He-like O vii lines, for example, are 21.6 Å \((r)\), 21.8 Å \((i)\), and 22.1 Å \((f)\) (Gabriel and Jordan, 1969). This proximity in wavelength means that ratios of these lines can be easily observed by a single instrument with essentially no calibration problems.

For optically thin emission, the two ratios, \(R = f/i\) and \(G = (f + i)/r\), are diagnostics of density and temperature respectively, where \(f\), \(i\), and \(r\) refer observationally to the frequency integrated line fluxes. The O vii ion is of particular interest in this regard for coronal conditions in both the quiet Sun and active regions, and much work has been done to calculate theoretically these O vii line ratios (Keenan et al., 1984; Doyle et al., 1983; cf. also the earlier calculations of Acton and Brown, 1978; Gabriel and Jordan, 1969).

Using the atomic data in Acton (1978) and the ionization equilibrium calculations of Jordan (1969) we find that for typical quiet Sun conditions \((T \approx 2 \times 10^6 \text{ K}; n_e \approx 2 \times 10^8 \text{ cm}^{-3})\) the O vii resonance line optical depth is \(\tau \approx 1\) for path lengths, \(s \approx 1 \text{ R}_\odot\). For active regions (typically \(T \approx 3 \times 10^6 \text{ K}; n_e \approx 2 \times 10^9 \text{ cm}^{-3}\)), the optical depth per unit path length is almost the same; the lower relative abundance of O vii at higher temperatures offsets the higher density in active regions resulting in the same volume opacity. The intersystem and forbidden lines remain optically thin, hence the ratio, \(G\), should manifest optical depth effects due to single scattering of radiation in the resonance line.

This effect was first observed by Acton and Catura (1976), again for O vii; more recently McKenzie and Landecker (1982) report modifications of \(G\) due to resonance scattering for both O vii and Ne ix.

3. Radiative Transfer Model

The geometry of the situation is shown in Figure 1. \(R\) is the solar radius, \(H\) is the coronal height, and \(r\) is the radial variable of integration along the line-of-sight. We are interested in finding the emergent intensity, \(I\), which is the sum of the direct emission from the column, \(R \leq r \leq R + H\), which we call \(I_d\), and the radiation scattered into the line-of-sight within the column, \(I_s\), originating from anywhere else in the corona. The emission per unit volume is \(\varepsilon\) and the cross section for scattering per unit volume is \(\kappa\), and both of these are assumed to be uniform throughout the corona.

For resonance scattering the redistribution phase function is the same as for Rayleigh scattering (Acton, 1978),

\[
\psi(\theta) = \frac{3}{4}(1 + \cos^2 \theta).
\]

In the single-scattering approximation, two effects take place: (1) emission originating in the line-of-sight column is reduced by scattering out of the line-of-sight; (2) some fraction of the emission originating elsewhere in the corona is scattered into the line-of-sight in the column. This component is also subject to attenuation by scattering out of