SURFACE REYNOLDS STRESSES DETERMINED FROM THE ANALYSIS OF FACULAR MOTIONS AND THE MAINTENANCE OF THE SUN'S DIFFERENTIAL ROTATION

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(Received 13 October; in revised form 28 November, 1975)

Abstract. From the analysis of the motions of faculae for a four year period from 1967 to 1970, the latitudinal angular momentum transport by Reynolds stresses at the Sun's surface is calculated. The result agrees fairly well with the one obtained by Ward analyzing spot motions. A comparison with theory suggests that the Sun's differential rotation could be maintained only by Reynolds stresses at the surface. The implications of this result are discussed.

1. Introduction

In this paper we shall emphasize one point of the very complex problem of the Sun's differential rotation: 'Are the Reynolds stresses in the surface layers of the solar convection zone important in maintaining the differential rotation?' The analysis of the interaction of the global convection with rotation (Busse, 1970; Durney, 1970, 1971; Yoshimura and Kato, 1971; Gilman, 1972) shows that an equatorial acceleration can be obtained due to the angular momentum transport towards the equator by Reynolds stresses. In this case the effect of Coriolis forces on a meridian circulation is negligible.

However, other authors (Weiss, 1964; Osaki, 1970; Durney and Roxburgh, 1971) suggest that due to the stabilising effect of rotation on convection, a pole-equator temperature difference arises which in turn drives a meridional circulation giving rise to a differential rotation by the effect of Coriolis forces. This problem is also connected with the problem of the flux difference between poles and equator. For a complete discussion of these arguments we refer to the recent works of Durney (1974a, b), Gierasch (1974) and especially to the review paper by Durney (1976) on the theories of solar rotation.

Here we only attempt to compare the latitudinal momentum transport by Reynolds stresses, as deduced from the motions of observable features in the Sun, with the term which allows for the angular momentum transport by Reynolds stresses in the equation for the angular velocity.

2. Data Analysis

Extensive work on mass motions in the Sun's surface layers, using K and Hα faculae as tracers, is in progress at the Catania Astrophysical Observatory (Godoli
et al., 1976). This work is based on Catania solar patrol data published since 1967 in the yearly bulletin ‘Solar Observation made at Catania Astrophysical Observatory’. At the present moment the data analysis is limited to a 4 yr period from 1967 to 1970. K spectroheliograms are taken daily. Faculae are classified on the basis of their compactness from 1 (very small) to 4 (only single facula well defined in its contours). For each facula the observation time, the heliographic position of the geometrical center of gravity, the projected \( A_p \) and corrected \( A_c \) areas are given. It is assumed that \( A_c = \frac{1}{2} A_p \sec h \), where \( h \) is the heliocentric angle.

Since we are interested in the latitudinal transport by Reynolds stresses we must look for the deviations from the mean velocity components along the parallels \( u_\varphi = u_\varphi - \langle u_\varphi \rangle \) and along the meridians \( u_\theta = u_\theta - \langle u_\theta \rangle \), where the brackets denote space-time averages and \( \varphi \) and \( \theta \) are the longitude and polar angle, respectively. As is well known the averages \( \langle u_\varphi' u_\theta' \rangle \) represent the latitudinal momentum transport by Reynolds stresses (see for example McCormak and Crane, 1973). If we take \( u_\varphi >0 \) in the direction of the mean motion and \( u_\theta >0 \) towards the equator in both hemispheres, then \( \langle u_\varphi' u_\theta' \rangle >0 \) means that momentum is transported towards the equator: namely, on the average faculae with \( u_\varphi - \langle u_\varphi \rangle >0 \) move towards the equator and faculae with \( u_\varphi - \langle u_\varphi \rangle <0 \) move towards the poles. According to Starr and Gilman (1965) the integral

\[
I = \int_0^{\pi/2} \langle u_\varphi' u_\theta' \rangle \sin^3 \theta \cos \theta \, d\theta - \int_0^\pi \langle u_\varphi' u_\theta' \rangle \sin^3 \theta \cos \theta \, d\theta
\]

represents the total angular momentum transport, appropriate dimensional factors being excepted. When \( I >0 \) the transport is towards the equator. For a given latitude strip (5 degrees) we calculated the mean velocities \( \langle u_\varphi \rangle \) and \( \langle u_\theta \rangle \) and the daily deviations \( u_\varphi' \) and \( u_\theta' \) for the 4 yr period. We then calculated the averages \( \langle u_\varphi' u_\theta' \rangle \) for each latitude strip relative to the period under consideration and finally the integral \( I \). In Table I we give the integral \( I \) (cm\(^2\) s\(^{-2}\)) as calculated for faculae of class four compactness and all classes of compactness, considering faculae of corrected areas \( A_c < 2 \times 10^3 \, E \), \( A_c < 5 \times 10^3 \, E \) and all areas, where \( E = 10^{-6} \) of solar hemisphere.

<table>
<thead>
<tr>
<th>Compactness</th>
<th>( A_c &lt; 2 \times 10^3 , E )</th>
<th>( &lt; 5 \times 10^3 , E )</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( 10^7 )</td>
<td>( 3 \times 10^7 )</td>
<td>( 4 \times 10^7 )</td>
</tr>
<tr>
<td>all</td>
<td>( 10^7 )</td>
<td>( 2 \times 10^7 )</td>
<td>( 4 \times 10^7 )</td>
</tr>
</tbody>
</table>

The number of phenomena we analyzed ranges from about 1000 in the most restrictive case \( (A_c < 2 \times 10^3 \, E, \text{compactness} = 4) \) to about 9000 when we considered all faculae. As is possible to deduce from Table I, we have \( I >0 \) in all cases and the values obtained are of the same order. The errors in the determination of the velocities are essentially due to the error in the determination of the