ON THE SOLUTION OF THE TRANSFER EQUATION
SYSTEM FOR A NON-SCATTERING MEDIUM
WITH A HOMOGENEOUS MAGNETIC FIELD

(Research Note)

J. M. KATZ*
Siberian Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, Irkutsk, U.S.S.R.

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The exact solution of the boundary problem for the system of three transfer equations with Unno's absorption matrix was obtained by Kjeldseth Moe (1968) who studied the formation of Fraunhofer lines in a magnetoactive atmosphere.

This paper describes another method, which gives the exact solution of the boundary problem for a transfer equation system of any order with an arbitrary absorption matrix, able however to satisfy condition (4).

The main assumptions are the same as in Moe:
(1) The atmosphere is supposed to be plane-parallel.
(2) In the atmosphere the magnetic field is stationary and homogeneous.
(3) The light scattering is omitted.

According to these assumptions we have the following boundary problem for the polarized radiation field:

\[ \frac{dI(\tau, p)}{d\tau} = A(\tau, p) I(\tau, p) - B(\tau, p), \quad \tau > 0; \]  \hspace{1cm} (1)

\[ I(0, p) = 0 \]  \hspace{1cm} at \hspace{1cm} \mu < 0; \hspace{1cm} (2)

\[ \lim_{\tau \to \infty} e^{-ct} I(\tau, p) = 0, \]  \hspace{1cm} \hspace{1cm} c = \text{const} > 0. \hspace{1cm} (3)

Here \( I = (I_1, \ldots, I_N) \) is the Stokes vector-function; \( A = \|A_{ij}\| \) is the absorption matrix; \( B = (B_1, \ldots, B_N) \) is the vector of the primary sources of radiation. The parameter \( \mu \) characterizes the direction of radiation propagation. The vector-parameter \( p \) can have such components as the angle between the direction of the radiation propagation and the vector of the magnetic field, the strength of magnetic field, the distance between the fixed point on the contour and the center of the line, the constant of damping, etc. The order of the transfer equation system \( N \) is determined by the concrete choice of the Stokes vector (in different problems \( N = 1, 2, 3, 4 \)).

* On leave from the Computing Center of Peter Stučka Latvian State University, Riga, U.S.S.R.
In the theory of Fraunhofer lines for homogeneous magnetic field and arbitrary $N$ the absorption matrix can be represented in the form:

$$A(\tau, p) = E + \eta(\tau) a(p)$$  \hspace{1cm} (4)

where $E$ is the unit matrix. A spatial part of the absorption matrix $\eta(\tau)$ is a scalar function determined by the choice of the concrete stellar atmosphere model.

$a(p)$ denotes the frequency-angular part of the absorption matrix.

Moe's method as well as the proposed one is based on the separation of the absorption matrix into two parts: the frequency-angular matrix and the scalar spatial part.

Now we proceed to the solving of the boundary problem (1)–(3). If we denote

$$I(\tau, p) = e^{\tau/\mu} U(\tau, p),$$ \hspace{1cm} (5)

then

$$I(0, p) = U(0, p).$$ \hspace{1cm} (6)

According to (2) we require

$$U(0, p) = 0 \text{ at } \mu < 0.$$ \hspace{1cm} (7)

Substituting (5) into (1) we get

$$\mu \frac{dU(\tau, p)}{d\tau} = \eta(\tau) a(p) U(\tau, p) - e^{-\tau/\mu} B(\tau, p).$$ \hspace{1cm} (8)

In the same way as in Kjeldseth Moe (1968) we introduce a new variable

$$z = z(\tau) = \int_0^\tau \eta(s) ds.$$ \hspace{1cm} (9)

Then Equation (8) becomes

$$\mu \frac{du(z, p)}{dz} = a(p) u(z, p) - b(z, p),$$ \hspace{1cm} (10)

where

$$u(z, p) = U(\tau(z), p),$$ \hspace{1cm} (11)

$$u(0, p) = U(0, p),$$ \hspace{1cm} (12)

$$b(z, p) = \frac{e^{-\tau(z)/\mu}}{\eta(\tau(z))} B(\tau(z), p).$$ \hspace{1cm} (13)

Using (7) and (12) one can deduce

$$u(0, p) = 0 \text{ at } \mu < 0.$$ \hspace{1cm} (14)

According to the general theory of the linear differential equations the solution of the