A Pulsed NMR Study of $^3$He-$^4$He Solutions

J. R. Owers-Bradley, D. R. Wightman, A. Child, A. Bedford, and R. M. Bowley

Department of Physics, University of Nottingham, Nottingham, England

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We report a study of the Leggett-Rice effect in dilute $^3$He-$^4$He solutions at a Larmor frequency of 71 MHz. We have measured the height and phase of the spin echoes following 2-pulse NMR sequences of three solutions with $^3$He concentrations of 0.56%, 1.4%, and 4.0%. The theory of Leggett fits the data extremely well and yields values of the parameter $\mu M_0$ and the transverse diffusion coefficient $D$. We are also able to evaluate $\mu M_0$ and $D$ by analyzing multiple spin echoes and find good consistency between the two methods. From values of $\lambda/(1 + F_0^2)$ at various concentrations, we are able to determine the scattering length for $^3$He quasiparticles, $a$, quite accurately. We find $a = -0.97 \pm 0.03 \AA$.

1. INTRODUCTION

Large changes in the transport properties of a dilute $^3$He-$^4$He solution are expected when the system becomes significantly spin polarized, something which can be achieved by applying a field of about 10 Tesla to a solution at temperatures below 10 mK. The viscosity, thermal conductivity, and longitudinal spin diffusion constant all increase because the mean free path increases on polarization.

The changes expected in the transverse spin diffusion constant, $D$, are not so obvious. For small polarizations the spin dynamics are governed by the Leggett–Rice effect: the precession of the spin current around the internal molecular field. Leggett derived the following equations for the normalized height $h(t_1)$ and the rf phase $\phi$ of the spin echo signal induced by the NMR pulse sequence $\theta-t_1-180^\circ$ in the presence of magnetic field gradient, $G$.

$$ \frac{1}{2}(1 + \mu^2 M_0^2 \cos^2 \theta) \log_e(h(t_1)) + \frac{\mu^2 M_0^2}{2} \sin^2 \theta(h^2(t_1) - 1) $$

$$ = -\frac{1}{2} \gamma^2 G^2 Dt_1^3 $$

(1)
and

\[ \phi = -\mu M_0 \cos \theta \log(h(t_1)) \]  

(2)

We are able to fit the echo heights, \( h \), to Eq. (1) as a function of \( t_1 \) for various tipping angles, \( \theta \), and hence extract a value for \( \mu M_0 \), where \( \mu \) is the spin rotation parameter and \( M_0 \) is the magnetization. For degenerate Fermi systems, the quantity \( \mu M_0 \) is sometimes written as \( \omega_0 \lambda \tau \), where \( \omega_0 \) is the Larmor frequency, \( \tau \) is the transverse relaxation time and \( \lambda \) is a dimensionless parameter which depends on Landau parameters. \( \lambda \) contains details of the interaction between the particles.

Instead of measuring echo heights, we can measure the phase of the echo as a function of the height and extract \( \mu M_0 \) using Eq. (2), something which has not been done previously. This works best for smaller tipping angles—it fails completely for \( \theta \) near 90°.

The validity of Leggett's equation, that is Eq. (1), was first demonstrated by Corruccini \textit{et al.} \textsuperscript{4} in pure \( ^3 \)He and, subsequently, in dilute solutions. The concentrations of the solutions which they used were 5%, 6.4%, and 8.27% in magnetic fields of 0.24 and 1.13 Tesla—the spin polarization was always less than 1%. They were able to measure \( D \) for each solution, but unfortunately, it was only possible to obtain a value of \( \lambda \) for the 6.4% solution, and even then there was a large error in \( \lambda \) and no knowledge of the sign.

The obvious question is this: what happens to the Leggett–Rice effect as the spin polarization increases? In more detail we can ask the following. Does Leggett's equation still work? Are there other effects which cause it to break down? For example, there is heating of the system as the magnetic energy, \( \hbar \omega_0 \), is released, causing the relaxation time to vary with time. How does the transverse relaxation time vary with spin polarization and with temperature? Are there any experimental difficulties which need to be overcome?

The first spin echo experiments on a highly polarized solution were carried out by Gully and Mullin.\textsuperscript{5} They opted for a low concentration of 0.037% and a field of 8.9 Tesla, giving a spin polarization in excess of 30% at their lowest temperature. They were able to measure \( D \) and also evaluate \( \mu^2 M_0^2 \) as a function of temperature by fitting Eq. (1) to their echo heights for different echo times and tipping angles. If we assume \( \lambda \) is reasonably temperature independent, then \( \omega_0 \lambda \tau \) should increase as the temperature is reduced over the temperature region they studied. Instead Gully and Mullin observed a maximum in \( \mu^2 M_0^2 \) at around 25 mK. This result is unexpected and has never been properly explained. Is it a real effect or was there an error in their experiment?

Recent work by Candela \textit{et al.}\textsuperscript{6,7} who generated spin wave resonances in a 0.18% and 0.0625% solutions at 8 Tesla, has shown that \( \mu M_0 \) does