Approval voting, Condorcet's principle, and runoff elections

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Abstract

Approval voting allows each voter to vote for as many candidates as he wishes in an election but not cast more than one vote for each candidate of whom he approves. If there is a strict Condorcet candidate — a candidate who defeats all others in pairwise contests — approval voting is shown to be the only nonranked voting system that is always able to elect the strict Condorcet candidate when voters use sincere admissible strategies. Moreover, if a strict Condorcet candidate must be elected under ordinary plurality voting when voters use admissible strategies, then he must also be elected under approval voting when voters use admissible strategies, but the converse does not hold.

The widely used plurality runoff method can also elect a strict Condorcet candidate when voters use admissible strategies on the first ballot, but some of these may have to be insincere to get the strict Condorcet candidate onto the runoff ballot. Furthermore, there is no case in which the strict Condorcet candidate is invariably elected under the plurality runoff method when voters use admissible first-ballot strategies. Thus, approval voting is superior to the plurality runoff method with respect to the Condorcet principle in its ability to elect the strict Condorcet candidate by sincere voting and in its ability to guarantee the election of the strict Condorcet candidate when voters use admissible strategies. In addition, approval voting is more efficient since it requires only one election and is probably less subject to strategic manipulation.

Approval voting in a multicandidate election allows each voter to vote for as many candidates as he wishes. A candidate receives one full vote from each person who votes for him regardless of how many other candidates that person votes for; the candidate with the most votes wins the election. We

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have noted elsewhere [1, 3] that approval voting is superior in several ways to all other single-ballot systems that do not ask voters to rank candidates. These other systems differ from approval voting by restricting the number of candidates that voters are allowed to vote for, the prime example being plurality (vote for one) voting.

A major reason for preferring approval voting to plurality and other single-ballot nonranked systems is its lack of any restriction on the number of candidates for whom an individual can vote. Thus, voters can express their "approval" for all candidates they find relatively acceptable, and they do not have to worry about abandoning their favorite candidates when such candidates have slim chances of winning. Approval voting also promotes sincere voting and discourages strategic or manipulative voting more than other single-ballot systems [1, 3] and thereby enhances the likelihood that a candidate who is acceptable to a large proportion of the electorate will be elected. Furthermore, approval voting is more likely to elect a Condorcet or majority candidate — one who is preferred by a majority to each other candidate — when one exists for certain types of situations [1].

Our purpose in the present paper is two-fold: (1) to present new results that reinforce the apparent superiority of approval voting over other single-ballot nonranked systems; and (2) to introduce runoff election systems into our comparative analysis. The generic class of runoff systems consists of voting procedures whose first ballots are similar to the ballots of the single-ballot nonranked systems; the second or runoff ballot is a simple majority ballot between the two candidates who receive the most first-ballot votes.

In the next two sections we describe the voting systems we shall analyze in more detail and specify our assumptions about voters' preferences, both between individual candidates and between what we shall refer to as outcomes of elections. In section 3 we define the notion of admissible voting strategies on the only (or first) ballot; for each combination of a voting system and preference order, we identify the set of strategies that are admissible (feasible and undominated) for that combination. We note that for most preference orders the set of admissible voting strategies for any ordinary (single-ballot) system is a proper subset of the set of admissible strategies for the corresponding runoff system.

Next, in sections 4 and 5 we use the admissibility results to determine the abilities of various systems to elect a strict Condorcet candidate when one exists. In the first of these two sections, we investigate the existence of admissible, and sincere admissible, strategies that will elect the Condorcet candidate. In the latter section we consider cases in which the Condorcet candidate is invariably elected when all voters use admissible strategies.

To reinforce our conclusion that ordinary approval voting is superior to other single-ballot nonranked systems, we argue that ordinary approval voting has several advantages over runoff systems. However, our comparison between ordinary approval voting and the popular runoff plurality system