Patterns in the effects of infectious diseases on population growth

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Abstract. An infectious disease may reduce or even stop the exponential growth of a population. We consider two very simple models for microparasitic and macroparasitic diseases, respectively, and study how the effect depends on a contact parameter $\kappa$. The results are presented as bifurcation diagrams involving several threshold values of $\kappa$. The precise form of the bifurcation diagram depends critically on a second parameter $\xi$, measuring the influence of the disease on the fertility of the hosts. A striking outcome of the analysis is that for certain ranges of parameter values bistable behaviour occurs: either the population grows exponentially or it oscillates periodically with large amplitude.

Key words: Epidemic – Population regulation – Threshold values for contact parameter – Bistable behaviour – Oscillations

1 Introduction

One of the aims of mathematical modelling in population dynamics is to find a causal relationship between phenomena and the underlying mechanisms. We want to understand how mechanisms, which act in and between the individuals of a population, determine size and structure of the population as a whole. This aim justifies to some extent the study of simple and consequently unrealistic models, where “unrealistic” means that they lack predictive value. Simple and mathematically tractable models in terms of a few variables allow us to develop some idea of the way in which properties of individuals (as incorporated in the model structure and summarized in parameters) influence the dynamical behaviour of the system. The investigation of the similarities and differences in dynamical behaviour of models which are identical in all but one structural component may yield insight in the influence of the biological mechanism described by this particular component.

In this paper we study the dynamics of two simple epidemic models. One of them is a model for microparasitic diseases of the $S$-$I$ type, the other one is a
model for a host-parasite system. We consider a population which grows exponentially in the absence of the disease. We are interested in the regulation problem: how does the disease affect the growth rate of its host population? So we look at a time scale at which the demographic processes as well as the disease transmission are important and we want to know what kind of dynamical behaviour can result from their interaction. Of course the answer depends on many parameters. Our strategy will be to look for patterns of changes in dynamical behaviour as a contact rate parameter, which measures the infective strength, is increased. Our results will, in the spirit of May and Anderson [1, 16] and Busenberg and van den Driesche [5], identify various thresholds for this contact rate parameter. Moreover, we will find that the pattern of changes of dynamical behaviour for diseases which have a strong influence on fertility is different from that for diseases which have no or only a minor influence on fertility. Among other things this clarifies in a much simpler context some observations of Kretzschmar [13] about subcritical bifurcations and the possibility of bistable behaviour.

For populations which possibly grow beyond every bound the use of strict mass action kinetics is questionable. Some authors dealing with models for STD's (sexually transmitted diseases) or worm diseases have introduced homogeneous models, i.e. models in which the total population size is but a scaling variable (see [7, 8, 9]). Such models allow for exponential solutions, as we will see in detail below. Of course homogeneous contact rates are debatable at low population sizes and hence for the exponential solutions to be meaningful it is required that the exponent is positive. Another class of models involves a contact rate which is like the strict law of mass action at low population densities, but essentially homogeneous for high population densities. Such a contact rate can be introduced as a phenomenological description (see Dietz [6]) or as resulting from application of a time scale argument to a free living infective stage ([17], also see Subsect. 2.2 below). The models now allow the existence of endemic steady states. One of our aims in this paper is to demonstrate how exponential solutions with positive exponents occur as asymptotic solutions of the second class of models and how they connect to the steady states when the contact rate parameter varies.

We will first introduce the models and explain the biological meaning of the parameters. Next we will summarize the results for the two models in biological terms. Finally we are going to give a unified formulation for the two systems of model equations which we then analyse mathematically. We conclude with some remarks about the biological interpretation of the results.

2 Modelling microparasitic and macroparasitic diseases

The distinction between micro- and macroparasites was introduced by Anderson and May in two articles published in Nature in 1972 [2, 17]. They classify diseases that are caused by a virus or by bacteria as microparasitic and diseases caused by helminths or arthropods as macroparasitic diseases. Epidemiologically the main difference is that in microparasitic diseases possible reinflection of an already infected individual plays no role in the disease dynamics, while in macroparasitic diseases the number of reinfections, or, more precisely, the number of parasites per host, has to be taken into account when modelling the disease.