A GRAPH-THEORETIC METHOD TO QUANTIFY THE AIRLINE ROUTE AUTHORITY

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ABSTRACT

A U.S. certificated passenger airline must be authorized by the Civil Aeronautics Board before it can schedule a route to serve a set of cities. The route authority is described in a number of legal statements on a Route Certificate, which specifies the routing restrictions by which the airline must abide in offering the service. For cost considerations, an airline is often interested in the shortest flight-time routes between city pairs. This paper introduces a graph-theoretic method to quantify the legal statements in a Certificate. Through straightforward matrix operations, all the authorized nonstop and multistop routes, including the shortest time routes, can be generated. The method is a convenient tool to help an airline to generate alternative network routes as demonstrated by a case study of American Airlines.

Introduction

In planning its network route structures, a U.S. certificated passenger airline has to abide by the route Authority certificate and economic regulation of the Civil Aeronautics Board (C.A.B.). A carrier has to be granted a "Certificate of Public Convenience and Necessity" before it can serve the cities concerned. The Certificate specifies in a number of legal statements if and how service may be offered between a set of airports. To expedite routing and scheduling analysis, there have been several attempts to quantify these statements of the Certificate into a more convenient representation. Trans World Airlines (TWA) devised a tabular representation of the route authorities the carrier holds between the cities in its route system (AW & ST, 1965). At Lockheed-California, Sobel (1969) described a scheme to store Route Certificate information systematically in the computer. In this paper, a conceptually much simpler, graph-theoretic way to quantify the C.A.B. Route Authority is presented. The approach will generate all the authorized routes in a network via straightforward matrix multiplications. In addition,
one can readily compute the shortest nonstop and multistop flight routes between a city pair in accordance with the C.A.B. authorization. This method will serve as a handy tool to help air carriers explore a number of possible alternative nonstop/multistop services between each city pair in their networks. The tool is particularly valuable in view of the current regulatory reform in the United States, which allows for more flexible market entries and exits (Chan and Ponder, 1978).

1. Configuring a Route Network

It is well known that network topologies can be represented algebraically as matrices (Avondo, 1962). One of such matrices is the “Contiguity matrix,” in which a connection by a link or arc is recorded as a corresponding node pair in a square matrix with a dimension equal to the number of nodes in the network. The contiguity matrix for the network of Fig. 1 is shown below:

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & (1,5) \\
0 & 0 & (2,3) & 0 & (2,5) \\
0 & 0 & 0 & (3,4) & (3,5) \\
(4,1) & 0 & 0 & 0 & 0 \\
0 & (5,2) & (5,3) & (5,4) & 0
\end{bmatrix}
\]

Fig. 1. An Example Network.

The contiguity matrix has the property that when raised to its second power, all the one-stop routes can be generated (Bellman et al., 1970). In the example below, only entry 3–4, instead of the entire matrix, is shown for the sake of clarity: