0. Introduction

The theory of color symmetry deals with the problem of how to add colors to different portions of a symmetric design, pattern, crystal, etc., in such a way that the elements of its symmetry group $G$ will each be associated consistently with a permutation of the colors; e.g., if $g$ takes any region colored red onto one colored yellow, it must take all red regions onto yellow ones. (See, for example, references [1], [2], [3], and [5].) It often occurs, however, that a design may have some local symmetries which do not belong to $G$; that is, it may contain many copies of symmetric motifs whose symmetries are not symmetries of the overall pattern. For example, in Figure 1 we have a pattern whose overall symmetry group is $pm$ but which contains an infinite collection of pentagons each having symmetry group $D_5$. Figure 2 shows a frieze diagram whose symmetry group ($1pm$) consists of translations and reflections in vertical axes, but it contains a collection of equilateral triangles which individually have symmetry group $D_3$.

The usual approach to such designs is to ignore any extra local symmetries and simply consider the overall symmetry group $G$. In this article we consider the problem of how to color a design in such a manner that the motifs considered separately are also each colored symmetrically.
1. BACKGROUND

The basic theory for symmetric coloring is explained in [3]. Generally, for a design with symmetry group $G$, a sequence $\{A_i\}$ of disjoint fundamental regions is selected such that for any pair $A_1, A_2$ there is a unique $g \in G$ with $A_1 g = A_2$. If one of them, $\Omega$, is selected as the 'starting region', then the regions may be labeled by the elements of the group, identifying $\Omega g$ with $g$. We shall be writing our transformations on the right and composing from left to right. Let $\mu$ be a transitive permutation representation of $G$ on a set of colors $S = \{1, 2, \ldots, n\}$. We shall use the notation $\mu(g)$ or $\mu_g$ for the permutation assigned to $g$. A coloring of the design is obtained by picking a color $j \in S$ and assigning $j$ to $\Omega$. Then to each region $\Omega g$ assign the color $(j)\mu_g$. If $H$ is the stabilizer subgroup of the color 1 and $\mu(x_i)$ takes 1 to $i$ for $i = 1, \ldots, n$, then this coloring corresponds to the 'biset decomposition' $x_j^{-1}Hx_1 \cup x_j^{-1}Hx_2 \cup \cdots \cup x_j^{-1}Hx_n$ of $G$ (see [3, Section 2]). Denote this coloring $[x^{-1}_jH]$. Two such colorings $[x_j^{-1}H]$ and $[x_k^{-1}H]$ are equivalent precisely if $x_k x_j^{-1} \in N_G(H)$, where $N_G(H)$ denotes the normalizer of the subgroup $H$ in $G$ (see [4, p. 2037]). Note that, in general, two symmetric colorings are called equivalent if one may be transformed to the other by a relabelling of the colors.

If several associated sequences of fundamental regions are selected (for example, by subdividing the regions), then a compound coloring (of 'type I') associated with $\mu$ may be obtained by using different biset decompositions for each sequence. If different transitive permutation representations (on disjoint sets of colors) are used for different sequences, we get a compound coloring of 'type II' which essentially involves an intransitive representation of $G$ (see [3, Section 3]).

2. GENERAL THEORY OF LOCAL COLOR SYMMETRY

Assume then that one has a pattern, design or structure with overall symmetry group $G$ and included in the design is a collection $\mathcal{M}$ of disjoint congruent