"Negative roughness" and polymer drag reduction

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Abstract. Based on an analogy to the Colebrook-White equation, a technique has been developed to allow polymer-solution extrapolation or "scaling" from one pipe size to another at constant values of Δβ. Each experimental data point can be transferred to a new pipe size by a simple, pocket-calculator method which preserves the experimental value of AB exactly. Thus scaling can be easily accomplished, without resorting to iteration or graphical techniques. The "negative-roughness" idea can also explain the loss of AB or drag reduction with increasing flow velocity.

List of symbols

A, B constants in velocity profile equation
Δβ constants corresponding to roughness (actual or negative)
D pipe diameter, m
k, height of sand-type roughness, m
N nondimensional negative roughness parameter
Re Reynolds number, UD/v
U average velocity in pipe, m/sec
u* local velocity in pipe, nondimensionalized with u*
u* friction velocity, m/sec
y radial distance from pipe wall, m
y* nondimensional distance from wall, yu*/v
Δ Darcy friction factor
v kinematic viscosity, m²/sec

Subscripts
1 experimental data
2 predicted

1 Introduction

In studying the effect of pipe roughness on the friction factor, it is usual to refer to a change in the velocity profile which includes a term Δβ. Thus the velocity profile becomes

_u* = A ln y* + B - Δβ_

where A = 2.5 and B = 5.5, approximately, and Δβ is the roughness contribution.

Meyer (1966), Elata et al. (1966), Fabula et al. (1966), and also Granville (1968), introduced the Δβ idea applied to polymer drag reduction, where now

_u* = 2.5 ln y* + 5.5 + Δβ_

and the Δβ concept has become an important part of the literature. We may look on the +Δβ term as a "negative roughness" in contrast to the -Δβ term shown, for example in Schlichting (1979), to be applicable to actual roughness.

However the Δβ idea is seldom if ever used in actual pipe roughness problems, since the roughness can be estimated and used as a parameter k/D on the friction factor-Reynolds number chart (Moody (1944)). Depending on the magnitude of the roughness (or better, the k/D ratio) it is found that the friction factor is constant over a considerable Reynolds number range. The behavior of λ over the complete range of turbulent Reynolds numbers is described by the Colebrook-White (1937) equation

1/√λ = -2 log \left[ 2.51 \frac{Re \sqrt{λ}}{k_s} + 3.7 D \right].

Comparing this with the Δβ relationship

1/√λ = 2 \log \left[ \frac{Re \sqrt{λ}}{2.51} \right] - \frac{Δβ}{\sqrt{8}}

we see that

Δβ = 2\sqrt{8} \left[ 2.51 \log \left[ \frac{Re \sqrt{λ}}{2.51} \right] + \log \left[ \frac{2.51}{Re \sqrt{λ}} + \frac{k_s}{3.7 D} \right] \right]

so that it becomes apparent that Δβ, while appearing attractive as a parameter on the u* - y* diagram shown in Figure 1, is awkward to apply in the actual roughness situation. Instead the Moody diagram (a portion of which is sketched in Figure 2) or the Colebrook-White equation is used.

If +Δβ is regarded as a negative roughness (Fig. 1), it should be possible to construct an analog to both the Colebrook-White equation and the Moody diagram using a neg-
ative roughness parameter, $N$. Thus we suggest

$$\frac{1}{\sqrt{\lambda}} = 2 \log \left[ \frac{Re \sqrt{\lambda}}{2.51} + N \right]$$

where now $N$ encapsulates the "negative roughness" achieved by a given polymer solution at a given concentration in a given pipe. Note that at larger values of $N$, the value of $\lambda$ is constant over a wide range of Reynolds numbers, and that lines of constant $N$ appear as analogs to the constant $k_r/D$ lines of the Moody diagram (Fig. 2).

Comparing our analog equation with the $AB$ equation for polymer drag reduction

$$\frac{1}{\sqrt{\lambda}} = 2 \log \left[ \frac{Re \sqrt{\lambda}}{2.51} \right] + \frac{\Delta B}{\log \lambda}$$

we see that now

$$\Delta B = 2 \sqrt{8} \left( \log \left[ \frac{Re \sqrt{\lambda}}{2.51} + N \right] - \log \left[ \frac{Re \sqrt{\lambda}}{2.51} \right] \right)$$

and the direct use of $\Delta B$ appears to be a rather complex approach to scaling from one polymer flow to a flow of a similar solution in a pipe of different diameter. Thus it is proposed to use the negative roughness parameter, $N$, in scaling method, just as $k_r/D$ is used for scaling actual pipe roughness.

2 A numerical demonstration

An example will show the method. Suppose we have the experimental data plotted in Figure 3 for a given polymer solution flowing in a pipe of diameter $D_1$. The data (Hoyt and Sellin (1988)) are for 4 ppm Separan AP302 in a 53 mm pipe. We ask for the performance achievable in a 106 mm pipe flowing the identical solution. Entering the analog equation with, say, the experimental values at minimum $2.21 = 0.0073$, we find

$$\frac{Re \sqrt{\lambda}}{2.51} = 711,307$$

In analogy to the $k_r/D$ procedure for actual roughness, we multiply the experimental value of the above quantity by $D_2/D_1$ to obtain a value for which

$$\frac{Re \sqrt{\lambda}}{2.51} = 2 \log \left[ \frac{D_2}{D_1} \left( \frac{Re_1 \sqrt{\lambda_1}}{2.51} + N \right) \right] = 2 \log \left[ \frac{106}{53} \right] (711,307)$$

and thus $\lambda_2 = 0.0066$.

Knowing $\lambda_1, \lambda_2$, and the Reynolds number at the experimental point ($Re_1 = 272,000$), and assuming that a given drag reduction will occur at the same friction velocity, $u^*$, regardless of pipe size, for a given polymer solution, we find the Reynolds number for the predicted point to be

$$Re_2 = Re_1 \sqrt{\frac{\lambda_1}{\lambda_2}} = 572,120$$

Comparing $\Delta B$ calculated from the original data with that of the predicted point shows that $\Delta B$ is unchanged in this procedure. This follows, since

$$\Delta B_1 = 2 \sqrt{8} \left( \log \left[ \frac{Re_1 \sqrt{\lambda_1}}{2.51} + N \right] - \log \left[ \frac{Re_1 \sqrt{\lambda_1}}{2.51} \right] \right)$$

2.00

0.50

0.00

-0.50

-1.00

-1.50

-2.00

Fig. 1. Sketch illustrating the $-\Delta B$ concept for actual pipe roughness and $+\Delta B$ for "negative roughness"