Digital phase-stepping holographic interferometry in measuring 2-D density fields

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Abstract. This paper presents a holographic interferometer technique for measuring transparent (2-D or quasi 2-D) density fields. To be able to study the realization of such a field at a certain moment of time, the field is “frozen” on a holographic plate. During the reconstruction of the density field from the hologram the length of the path traversed by the reconstruction beam is diminished in equal steps by applying a computer controlled voltage to a piezo-electric crystal that translates a mirror. Four phase-stepped interferograms resulting from this pathlength variation are digitized and serve as input to an algorithm for computing the phase surface. The method is illustrated by measuring the basically 2-D density field existing around a heated horizontal cylinder in free convection.

List of symbols

- $\lambda$: wavelength
- $x, y$: cartesian coordinate system
- $\Phi$: phase
- $\Delta \Phi$: phase step
- $K$: Gladstone-Dale constant
- $L$: width of 2-D density field
- $\varrho$: density
- $\varrho_0$: density at reference conditions
- $I_{1}, I_{2}, I_{3}$: recorded interferograms
- $I_{mod}$: modulation intensity
- $I_{bias}$: bias intensity
- $N$: numerator determining $\tan \{ \Phi (x, y) \}$
- $D$: denominator determining $\tan \{ \Phi (x, y) \}$
- $Q$: dimensionless temperature
- $T$: temperature
- $T_c$: temperature of cylinder
- $T_e$: temperature of environment
- $p$: pressure
- $R$: gas constant

1 Introduction

Unlike the conventional evaluation of (holographic) interference patterns (Becker 1985), where a single interferogram allows the determination of the phase surface by interpolation between the ordered positions of the fringe minima and fringe maxima in the interferogram (which are lines of constant phase), digital phase-stepping (holographic) interferometry calculates the phase in each point $(x, y)$ of the detector array and hence no interpolation techniques are required to find a full definition of the phase surface. The measured phase $\Phi (x, y)$ is related to the density field $\varrho (x, y)$ in the following way:

$$\frac{\lambda}{2 \pi} \Phi (x, y) = KL \left\{ \varrho (x, y) - \varrho_0 \right\}$$

where $\lambda$ is the wavelength of the applied laser light, $K$ is the Gladstone-Dale constant, $L$ is the width of the (2-D) density field in the direction of light propagation and $\varrho_0$ is the density at reference conditions. With $\lambda = 514 \text{ nm}$, $K = 0.227 \times 10^{-3} \text{ m}^3/\text{kg}$ and $L = 0.15 \text{ m}$ the density difference between two points on the detector array with a phase difference of $2 \pi$ radians is equal to: $1.51 \times 10^{-2} \text{ kg/m}^3$.

2 Experimental set-up

Figure 1 shows the experimental set-up, which consists of a holographic interferometer connected to a computer system. A CW Argon ion laser with a maximum output at single frequency operation of 1 W is placed on top of the interferometer. The field under study is the density field existing around a heated horizontal cylinder in free convection. The hollow cylinder is filled with hot water. In order to obtain a uniform temperature at the cylinder's surface, the cylinder was made of brass, ensuring good heat conduction. The cylinder is placed between the left and the right part of the interferometer, so that the collimated scene beam will propagate parallel to the cylinder's surface. Unlike the procedure followed by Watt and Vest (1987), only the field induced by the heated cylinder is recorded on the hologram. In this way no attention has to be paid to overlapping cross reconstructions, a phenomenon which is encountered when working with two reference beams. By freezing the field on the holographic plate, undesirable disturbances in it (caused e.g. by draughts) occurring at the time of interferogram formation are avoided. The interferometer is connected to a computer system to control the motion of a piezo-electrically driven mirror via a 12 bit D/A converter and so to achieve the...
3 Phase measurement

The basic equation describing the intensity distribution in an interferogram is given by:

\[ I(x, y) = I_{\text{bias}}(x, y) + I_{\text{mod}}(x, y) \cos \{ \Phi(x, y) + i \cdot \Delta \Phi \} \]  

(2)

where \( I_{\text{bias}}(x, y) \) is the bias intensity in the interferogram, \( I_{\text{mod}}(x, y) \) is the modulation intensity, \( \Phi(x, y) \) is the phase as defined before, \( \Delta \Phi \) is the applied phase step between successive interferograms and finally \( i \) is an integer number indicating the order of the phase steps. Accordingly, the intensity in an interferogram has a lower limit equal to \( I_{\text{bias}}(x, y) - I_{\text{mod}}(x, y) \) and the upper limit equal to \( I_{\text{bias}}(x, y) + I_{\text{mod}}(x, y) \). It must be remarked that in the sequel of this paper the dependence on the spatial coordinates \( x \) and \( y \) will be omitted in some places for notational reasons.

If four interferograms, \( I_0, \ldots, I_3 \), are digitized then the phase surface can be computed from them by applying an algorithm that was first introduced by Carré (1966):

\[ \tan \{ \Phi(x, y) \} = \frac{\sqrt{[(I_0 - I_3) + (I_1 - I_2)] \cdot [3(I_1 - I_2) - (I_0 - I_3)]}}{|I_1 + I_2 - (I_0 + I_3)|} = \frac{N(x, y)}{D(x, y)}. \]  

(3)

In the derivation of this algorithm the exact value of the phase step \( \Delta \Phi \) is not important and hence the algorithm is independent of the amount of mirror shift induced by the piezo crystal (Cheng and Wyant 1985). This illustrates the great advantage of this algorithm over other phase measuring algorithms, which all need a calibration of the piezo crystal first (Creath 1985) and hence strongly depend on the repeatability of the phase step. Another advantage is that the phase step is allowed to differ from point to point in the image, as may occur due to tilt of the mirror when translated by the piezo crystal or when a phase step is introduced in a slightly diverging or converging reconstruction beam, enabling this algorithm to compute the phase surface more accurately than other phase-measuring algorithms.

A separate evaluation of the numerator and the denominator determining \( \tan \{ \Phi(x, y) \} \) in Eq. (3) yields:

\[ N(x, y) = C(x, y) \cdot |\sin \{ \Phi(x, y) \}| \]  

(4)

\[ D(x, y) = C(x, y) \cdot |\cos \{ \Phi(x, y) \}|. \]  

(5)

This indicates that, apart from a positive multiplicative factor \( C(x, y) \) which depends on the phase step \( \Delta \Phi \) and on the modulation intensity \( I_{\text{mod}}(x, y) \), the numerator and the denominator are equal to the absolute value of \( \sin \{ \Phi(x, y) \} \) and \( \cos \{ \Phi(x, y) \} \) respectively, so that \( \Phi(x, y) \) is defined on the interval \([0; \pi/2]\). To find the phase on the interval \([0; 2\pi]\), the numerator is supplied with the sign of \( \sin \{ \Phi(x, y) \} \) which can be obtained from the recorded interferograms in the following way:

\[ \text{sign} \{ (I_0 - I_3) + (I_1 - I_2) \} \]  

(6)

and the denominator is supplied with the sign of \( \cos \{ \Phi(x, y) \} \) which is given by the expression:

\[ \text{sign} \{ (I_1 + I_2 - (I_0 + I_3) \}. \]  

(7)

Consequently, the values of the modified fields \( N(x, y) \) and \( D(x, y) \) are pixelwise inserted in the 2-D phase lookup table depicted in Fig. 2 along the vertical axis and along the horizontal axis respectively to show up with the phase \( \Phi(x, y) \) modulo \( 2\pi \) radians. Finally, the \( 2\pi \)-jumps remain to be detected and to be removed. This process is called phase-unwrapping. It is executed in a conventional way by starting from the lower left corner of the image and unwrapping successive columns by adding an offset if a \( 2\pi \)-jump is encountered.

4 Results

Figure 3a–d shows four phase-stepped interferograms that were taken from a heated cylinder (diameter: 8.3 cm, length: