An Inelastic Stress-Strain Law for Elevated Temperature and Slowly Time Varying Loads*

ERHARD KREMPL

Mechanics Division, Rensselaer Polytechnic Institute, Troy, New York 12181

(Received in revised form January 3, 1972)

ABSTRACT

Highly stressed components of jet engines, gas and steam turbines and nuclear reactors are subjected to comparatively few cycles of local plastic deformation followed by creep and/or relaxation during rated conditions at elevated temperature.

The behavior of structural metallic materials under simulated service conditions for a uniaxial state of stress at elevated temperature is shown to be characterized by rate (frequency) sensitivity, creep, relaxation, cyclic hardening or softening, and the aftereffect. These phenomena and prior deformation history have a considerable effect on the subsequent deformation and fracture behavior.

A realistic stress analysis for elevated temperature and slowly time varying loads such as occur in low-cycle fatigue with hold-time has to consider these material properties so that the stresses can be computed as a function of time. None of the conventionally used descriptions of material behavior (Plasticity, Creep or Viscoelasticity) can reproduce all the important observed phenomena. A new approach is proposed which introduces the concept of a fading or partially fading memory of discrete past events. It is postulated that the material remembers at least the maximum magnitude of the stress during the past history as well as the sign of the stress during the last unloading. For the uniaxial case a first-order nonlinear differential equation is proposed which contains the memory parameters in the coefficient functions. A specific choice is made and it is shown that the aftereffect and the permanent set can be reproduced qualitatively.

1. Introduction

Modern machinery such as jet engines, airframes of high-speed aircraft, steam and gas turbines and nuclear reactors are required to perform a complex pattern of operating schedules. One prime requirement is the capability of rapidly coming up to operating conditions and of rapidly changing loads. Although there is a different time scale involved in the various types of machinery, a common problem arises due to the required operating schemes. The rapid changes will cause local plastic deformation which will give rise to residual stresses at rated conditions. Creep and relaxation processes will then occur and the stresses will be redistributed during that time. Finally, the machine is shut down. During the lifetime of a machine, the start-up – rated condition – shut-down cycle repeats itself from a few hundred to several thousand times. Cracking of highly stressed and vital components is experienced because of this type of operation and a considerable amount of experimental research has been conducted in the area of better understanding the behavior of structural metals under these conditions, which are generally referred to as low-cycle fatigue.

Most of the laboratory investigations which are intended to supply information on the material behavior under the above-stated conditions, are conducted under isothermal conditions near the maximum operating temperature of the particular structural material. It has been found that frequency (rate of straining) and the introduction of hold-time which will lead to periods of creep or relaxation, can have a significant influence on both the deformation and the fracture behavior of structural materials.

It is the objective of this paper to review the present methods of stress analysis relevant to

* Based on a paper presented at 36th Meeting, Propulsion and Energetics Panel, AGARD-NATO, Florence, Italy, September 1970.
366

E. Krempel

elevated temperature conditions. Then the laboratory data for low-cycle fatigue involving hold-time are considered and the salient features of these data are emphasized. None of the presently used stress-strain laws can give a realistic prediction of the variation of the stresses in hold-time conditions. An outline is presented on how to develop a realistic stress-strain law applicable for elevated temperature stress analysis under arbitrary time varying loads.

2. Current Low-Cycle Fatigue Reliability Analysis

Reliability analysis of machine components consists of two main parts. The determination of the stresses (strains) as a function of location and time, and a decision whether the computed stresses and strains are safe within a certain margin which will vary from machine to machine. A schematic of the reliability analysis procedure is given in Fig. 1. The relevancy of a procedure for reliability analysis hinges on how well the equations employed model the actual deformation and fracture behavior of the component under question.

![Figure 1. Schematic showing the various steps involved in low-cycle fatigue reliability analysis.](image)

In design stress analysis, small strain elasticity theory is mostly used in conjunction with closed form solutions for idealized geometries or finite-element computer solutions for actual geometries. The applicability of this method to problems of low-cycle fatigue rests on the assumption of "Elastic-Strain Invariance", [1]. This assumption asserts that "plasticity, if it occurs, does not affect the total strains, although it may have large effects on the stresses". It is clear that this condition cannot be fulfilled in every application. However, this method is used widely in design applications [2–5]. In this way nominal strain ranges are obtained. The