Some issues related to the topological aggregation of preferences

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Abstract. This paper deals with the topological approach to social choice theory initiated by Chichilnisky. We study several issues concerning the existence and uniqueness of Chichilnisky rules defined on preference spaces. We show that on topological vector spaces the only additive, anonymous, and unanimous aggregation n-rule is the convex mean. We study the case of infinite agents and show that an infinite Chichilnisky rule might be considered as the limit of rules for finitely many agents. Finally, we show that under some restrictions on the preference space, the existence of a Chichilnisky rule for every finite case implies the existence of a weak Chichilnisky rule for the infinite case.

1. Introduction

When dealing with the issues that appear in Social Choice theory, it seems natural to give some structure to the set of preferences X under consideration; a topological structure, for instance. On the other hand, the aggregation rule (a map \( \Phi: X^n \to X \), where \( X^n \) is the cartesian product of \( X \) n-times, n being the number of individuals) should satisfy a set of “mild” assumptions, such as the respect of unanimity and something that would capture the notion of continuity.

There exist several models to aggregate preferences, each of them based on a list of restrictions that are imposed on the set of preferences or on the aggregation rule. The model that is most common in the literature is the Arrowian one. Another well known model is that introduced by Chichilnisky, in which the set of preferences is a topological Hausdorff space and the aggregation rule must be continuous, anonymous, and unanimous (henceforth, Chichilnisky rule).

Using this model, Chichilnisky and Heal (1983) proved a very deep result: “A parafinite CW-complex preference space admits a n-Chichilnisky rule, for every \( n \in \mathbb{N} \), if and only if the space is contractible”.

The contractibility of the space of preferences is in fact a restriction of the domain of preferences that are admissible, which is weaker than those used in
the Arrowian model, such as single peaknessness (see Black 1948), to avoid Arrow's impossibility result.

The present paper will use the framework introduced by Chichilnisky (1980). We study several issues concerning the existence and uniqueness of Chichilnisky rules. Here, a \textit{n-Chichilnisky rule} will be a continuous, unanimous, and anonymous map \( \Phi : X^n \rightarrow X \).

When \( X \) is endowed with some additional structure, it seems natural to look for aggregation rules that preserve that structure. For instance, if an addition is defined on \( X \) (and, consequently, on \( X^n \)), a \textit{n-Chichilnisky} rule \( \Phi \) "preserving the addition" (i.e.: \( \Phi(x_1 + y_1, \ldots, x_n + y_n) = \Phi(x_1, \ldots, x_n) + \Phi(y_1, \ldots, y_n) \)) is called \textit{additive}.

With this idea in mind, we shall prove that on a topological vector space \( X \) the only additive Chichilnisky rule \( X^n \rightarrow X \) is the convex mean.

In the infinite case of rules \( \Phi : X^\infty \rightarrow X \), (being \( X^\infty = \prod_{i=1}^{\infty} X \), endowed with the product topology), the situation is quite different. We obtain an impossibility result:

"There is no additive infinite-Chichilnisky rule on a topological vector space". (The definition of \textit{infinite-Chichilnisky rule} is similar to that given for the case of "n" agents, extending the idea of anonymity).

We compare different kinds of Chichilnisky rules. The relationship studied between different kinds of rules is based on the behaviour of families of \( n \)-rules (a \( n \)-rule for each \( n \in \mathbb{N} \)) when a limit \( (n \rightarrow \infty) \) is computed. In this line, we show that an infinite Chichilnisky rule might be considered as a limit of \( n \)-rules.

Finally, we deal with the problem of the existence of infinite Chichilnisky rules, and show that finitely anonymous infinite rules always exist on certain compact preference spaces. Moreover, they are the limit of similar \( n \)-Chichilnisky rules (in fact, under some restrictions on the preference space, these rules do not vary on some infinitely rearranged sequences). Therefore the concept of Chichilnisky rule involves an idea of "robustness".

\section{Motivation}

Let \( X \) denote a Hausdorff (i.e. separated) topological space, called the \textit{space of preferences}.

A \textit{n-Chichilnisky rule} (see Chichilnisky 1980) on \( X \) is a map

\[
\Phi : X^n \rightarrow X
\]

\[
(x_1, \ldots, x_n) \mapsto \Phi(x_1, \ldots, x_n)
\]

satisfying:

(i) \textit{Continuity}: \( \Phi \) is a continuous map \( (X^n \) is endowed with the product topology).

\footnote{The "infinite case" that we will always consider through the paper is "countable and infinite". (That is: Product spaces with a "continuum" of components are not considered here.) To deal with a "continuous" case, a different approach, using probability measures on \( X \), seems to be more adequate. See, for instance, Tanguiane (1979) or Marley (1991).}