Abstract. A simple method for the computer-aided calibration of an X-probe is described. This method requires the X-probe to be pitched in the free-stream at several velocities. From the corresponding output voltages, a calibration look-up table can be generated. The technique requires fewer assumptions than traditional methods based on King's law.

1 Introduction

X-probes have been used extensively in the investigation of turbulence, since they can provide simultaneous measurements of two components of velocity. This allows the measurement of the Reynolds stress, \(-\rho u'v'\), where \(\rho\) is the density, and \(u'\) and \(v'\) are the velocity fluctuations in two perpendicular directions. X-probes have two sensors, either hot-film or hot-wire, which are usually oriented perpendicular to each other and at a 45° angle to the mean flow. In this orientation, each sensor is cooled differently depending on the direction of the local flow. For example, if the local flow is perpendicular to one sensor and parallel to the other, then the sensor perpendicular to the flow sees substantial cooling, while the sensor parallel to the flow sees minimal cooling. The effective velocity, \(U_{\text{eff}}\), measured by the sensor is the component of velocity normal to the sensor. Thus, if \(Q\) is the magnitude of the velocity vector and \(\beta\) is the angle between the normal to the sensor and the velocity vector, then the velocity measured by the sensor is \(U_{\text{eff}} = Q \cos(\beta)\). This "cosine law" assumes a sensor of infinite length and negligible cooling caused by the tangential component of velocity. An X-probe depends on the directional sensitivity of the two sensors to measure the perpendicular velocity fluctuations, \(u'\) and \(v'\).

Until recently, hot-wire and hot-film probes were calibrated by assuming forced convection cooling of a heated cylinder according to King's (1914) law. This is a non-linear relationship between the fluid velocity and the output voltage of the hot-wire anemometer requiring two calibration constants for each sensor. To calibrate an X-probe using King's law, the calibration constants are estimated for each sensor by measuring the voltage output of each sensor for several velocities with the probe oriented at a single angle with respect to the flow. When the sensor output is linearized, one component of velocity is proportional to the sum of the output voltage, while the other component is proportional to the difference of the output voltages. Then, the sum and difference can be electronically correlated to obtain \(u'v'\). This calibration procedure is described in detail by Bradshaw (1971).

Several problems are inherent in this calibration procedure. First, the effective velocity measured by the sensor needs to be corrected to account for the finite aspect ratio of the sensor and the effect of cooling caused by the velocity component parallel to the sensor. Champagne et al. (1967) give a correction factor that depends on the sensor length and the angle between the sensor and mean flow. Second, the calibration scheme does not expose the X-probe to the components of velocity that will actually be measured. The calibration is based on results derived from measurements made with the probe axis at a single angle with respect to the calibration flow. Thus, if the probe is calibrated so its axis is along the calibration velocity vector, then any velocity component that is not parallel to the probe axis can only be estimated from the cosine law. Third, the calculation of the individual velocity components from the voltage outputs is dependent on the assumption that the sensors are at a 90° angle to one another. Bradshaw (1971) estimates that the ratio of \(v'\) component sensitivity to \(u'\) component sensitivity for a single sensor changes by 10% when the angle of the sensor to the flow is changed from 45° to 48°. Finally, this method depends on electronic linearization and correlation. High-speed computerized data acquisition has made available the superior calibration technique described below.

The multiple angle calibration technique exposes the X-probe to different velocity components by orienting the X-probe at several different angles with respect to the calibration flow. This method requires the simultaneous measurement of the voltage output of each sensor, \(E_1\) and \(E_2\), as
the X-probe is pitched in the calibration flow. A unique voltage pair \((E_1, E_2)\) results for a given pitch angle, \(\gamma\), and calibration velocity, \(Q\) (Fig. 1). For each voltage pair, \((E_1, E_2)\), the pitch angle and calibration velocity can be used to determine the components of velocity, \(U = Q \cos \gamma\) and \(V = Q \sin \gamma\). The advantage of this technique is that the X-probe is directly calibrated to the components of velocity that will be measured. No assumptions need be made with respect to the calibration constants for King's law, the cosine law, correction factors, or linearization of the output. In addition, the angle between the sensors need not be exactly 90°, since the calibration scheme requires no assumptions about the sensor orientation.

Theoretically, an X-probe with perpendicular sensors should be calibrated through a range of pitch angles \(-45° \leq \gamma \leq 45°\), since at \(\gamma = \pm 45°\) one sensor is perpendicular to the flow while the other is parallel to the flow. Our experience indicates that the practical range of angles is approximately \(-30° \leq \gamma \leq 30°\). Johnson and Eckelmann (1984) found a similar range of useful pitch angles. They point out that interference of sensor supports and the effect of the tangential component of the velocity on cooling of the sensor cause this reduction in the meaningful range of pitch angles.

Several different numerical techniques have been used to implement the multiple angle calibration technique on a computer. Oster and Wygnanski (1982) determined two third-order polynomials, each in two variables \((E_1, E_2)\), to represent the magnitude of velocity, \(Q\), and the angle, \(\gamma\), respectively. The measured voltage pair, \((E_1, E_2)\), is simply substituted into each polynomial to find \(Q\) and \(\gamma\) from which \(U = Q \cos \gamma\) and \(V = Q \sin \gamma\). Using 66 calibration points, they accepted the calibration only if the errors in \(U\) and \(V\) were less than 1% and 2%, respectively.

Willmarth and Bogar (1977) were the first to use a multiple-angle X-probe calibration technique in conjunction with a look-up table. They recorded \(E_1, E_2, Q,\) and \(\gamma\) while pitching the X-probe and slowly changing the wind tunnel speed. From this data, they calculated an array of equidistant voltage pairs, \((E_1, E_2)\), with a unique \(U, V, \partial U/\partial E_1, \partial U/\partial E_2, \partial V/\partial E_1,\) and \(\partial V/\partial E_2\) corresponding to each voltage pair. These six values were used to calculate \(U\) and \(V\) given any voltage pair, \((E_1, E_2)\).

Johnson and Eckelmann (1984) presented another look-up table numerical technique which uses a second order Taylor expansion between calibration points. The technique requires the calculation of first and second derivatives of \(E_1\) and \(E_2\) with respect to \(Q\) and \(\gamma\). Using the Taylor expansion and Newton's iteration technique, the values for \(U\) and \(V\) can be determined given a measured voltage pair, \((E_1, E_2)\). Although this technique does not require any secondary interpolations, the technique requires the estimation and storage of 12 calibration voltages and partial derivatives for each calibration point.

In this paper, we outline a third type of look-up table numerical scheme of accomplish the calibration of an X-probe. The scheme involves the development of a Cartesian calibration grid in the \(E_1 - E_2\) plane given the calibration data for several calibration angles, \(\gamma\), at different flow velocities, \(Q\).

This numerical scheme is presently used by several turbulence researchers in similar forms and was first reported by Zilberman (1981). However, the technique has not been documented in the literature. The purpose of this paper is to provide a description of this method of X-probe calibration.

\section*{2 Calibration technique}

To initiate this calibration technique, the X-probe is pitched through angles from \(-30°\) to \(30°\) in the free stream of a wind tunnel flow. Because of the time for the wind tunnel to reach a steady-state after the velocity is changed, this is accomplished by setting the wind tunnel velocity and then pitching the X-probe through the full range of angles. Then the wind tunnel velocity is changed, and the X-probe is again pitched through the full range of angles. Ideally, this entire operation could be computer-controlled, so that the computer automatically sets the wind tunnel velocity, pitches the X-probe, and measures the voltage outputs \((E_1, E_2)\).

A sample of the resulting calibration data appears in Fig. 2a. Notice that each arc of raw data represents the voltage pairs, \((E_1, E_2)\), at the same calibration velocity and different pitch angles. The lowest velocity appears at the lower left part of the figure. Lines of constant pitch angle radiate from the lower left corner of the figure.

The lowest velocity that can be measured accurately is limited by the heat transfer from one sensor of the X-probe to the other sensor. To determine the minimum velocity at which the sensors interfere with one another, one sensor is turned on and off with the other remaining on. Below a certain velocity, the cycling of one sensor can be detected in the voltage output of the other sensor which remains on. We have found that for air velocities below 1.5 m/s, the heat transfer between individual sensors can be detected for an X-probe with a box size of 0.25 mm with 1.25 μm Pt-Rh wires. We recommend that this method be used to determine the minimum velocity which can be measured without heat transfer between the sensors before using any X-probe.