Vector field approximation: a computational paradigm for motor control and learning

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Abstract. Recent experiments in the spinalized frog (Bizzi et al. 1991) have shown that focal microstimulation of a site in the premotor layers in the lumbar grey matter of the spinal cord results in a field of forces acting on the frog's ankle and converging to a single equilibrium position. These experiments suggested that the neural circuits in the spinal cord are organized in a set of control modules that "store" a few limb postures in the form of convergent force fields acting on the limb's end-point. Here, we investigate how such postural modules can be combined by the central nervous system for generating and representing a wider repertoire of control patterns. Our work is related to some recent investigations by Poggio and Girosi (1990a, b) who have proposed to represent the task of learning scalar maps as a problem of surface approximation. Consistent both with this view and with our experimental findings in the spinal frog, we regard the issue of generating motor repertoires as a problem of vector-field approximation. To this end, we characterize the output of a control module as a "basis field" (Mussa-Ivaldi 1992), that is as the vectorial equivalent of a basis function. Our theoretical findings indicate that by combining basis fields, the central nervous system may achieve a number of goals such as (1) the generation of a wide repertoire of control patterns and (2) the representation of these control patterns with a set of coefficients that are invariant under coordinate transformations.

1 Introduction

In a recent set of experiments (Mussa-Ivaldi et al. 1990; Giszter et al. 1991; Bizzi et al. 1991) in collaboration with E. Bizzi, we have investigated the organization of the motor output in the spinal frog. Briefly, we stimulated with a microelectrode a number of sites in the premotor layers of the gray matter. For each stimulation site, we measured the isometric forces produced by the muscles of the leg at different ankle locations (Fig. 1). Three major results emerged from this study.

1. As shown in Fig. 1D, the microstimulation of a site in the premotor layers of the lumbar gray matter resulted in a convergent field of forces at the ipsilateral ankle. The force vector vanished at a single equilibrium point within the limb's workspace. We attributed this convergent pattern of forces to the balanced recruitment of a group of agonist and antagonist muscles acting on the hip and knee joint. Muscles are known to behave as tunable spring-like elements (Rack and Westbury 1969). It seems reasonable to expect the combination of the forces generated by a group of such elements to be a field of elastic forces acting on the ankle and converging to an equilibrium location.

2. Through microstimulation of different spinal interneuronal regions, we elicited different force fields with different equilibrium points.

3. The convergent field added vectorially. When we applied two simultaneous microstimulations to two different spinal sites we obtained a force field that was proportional to the vector sum of the fields generated by the independent stimulation of each site.

These results strongly suggest that the premotor circuitry in the frog's spinal cord is organized in a number of distinct modules. Each module implements a control law which corresponds to the specification of a limb's posture. This control law is expressed by a field of forces with a single equilibrium position. As one adopts this point of view, it is natural to ask if and how the central nervous system may combine a set of output fields for generating other postures as well as more complex motor behaviors.

In the following sections we consider how the vectorial summation of a few force fields can generate a variety of control patterns. We address this issue from the point of view of approximation theory. We begin by defining a component of a desired controller's behavior as a pattern of force vectors over a limb's configuration.
Fig. 1A–D. Force fields obtained from the microstimulation of the frog's spinal cord. A The workspace locations at which the ankle forces were measured are indicated by the black dots. B The ankle's workspace was partitioned into a set of non-overlapping triangles (A, B, C, ...). The tested locations shown in A are at the vertices of each triangle. The arrows at the same vertices represent the force vectors measured at different points and at the same latency from the onset of the stimulus. These are actual data. The dashed arrow within the triangle B was derived by linear interpolation from the vectors at the vertices of the same triangle. C Interpolated field. The equilibrium point is indicated by a filled circle. D Same field as C, without the interpolation triangles. (From Mussa-Ivaldi et al. 1990)

space. In a closely related investigation, Mussa-Ivaldi (1992) has shown that the relevant features of many patterns of vectors can be captured by representing a continuous field as a combination of basis fields. Basis fields are the vectorial counterpart of the local basis functions used for reconstructing a scalar map from a set of numerical examples (Powell 1987). Poggio and Girosi (1990a, b) have used the theory of basis functions to characterize the behavior of a wide class of neural networks performing multivariate association maps. In accordance with their views, here we suggest to use basis fields for representing the operation of a network of elementary control modules. A biological example of such an elementary control module may be established by the pattern of connections between a spinal interneuron and a set of motoneurons innervating a group of different muscles. This pattern of connections between spinal interneurons and spring-like muscles has been suggested to be the underlying reason for the observation of convergent force fields after stimulation of the premotor layers in the frog's spinal cord (Bizzi et al. 1991).

More formally, we consider a network of control modules implementing mechanical behaviors which are described by basis fields. The total output of such a network is given by the vectorial summation of these basis fields. Our findings indicate that by combining basis fields a distributed control system may achieve a number of important goals such as (1) the generation of a wide repertoire of control patterns and (2) the representation of these patterns with a set of coefficients that are invariant under coordinate transformations.

In a sense, we present an extension of the equilibrium-point hypothesis (Feldman 1966; Bizzi et al. 1984; Hogan 1984) to a broader context. According to the equilibrium-point hypothesis, the central nervous system generates and represents the movement of a limb as a temporal sequence of equilibrium positions. One may observe that the notion of equilibrium position corresponds to a particular feature of a field of forces: a stable equilibrium position is a point in space surrounded by a pattern of attractive forces. Here we consider how a variety of force patterns, including (but not limited to) such stable equilibria can be obtained by superimposing the outputs of simple control modules. In particular, our results indicate that the pattern of output forces corresponding to the implementation of an ideal force controller can be generated by combining a number of equilibrium-point controllers. This finding is important because it shows that motor tasks as diverse as moving a limb and manipulating objects can be reduced to a single computational framework based upon the vectorial combination of convergent force fields.

2 Control, planning and vector-field approximation

Let us consider a simplified system consisting of a set of K independent control modules acting in parallel upon the muscles of a multi-joint limb. Each control module establishes the viscoelastic properties of a group of muscles. As a result, each module generates at the interface between the limb and the environment a force-field

\[ F^i = \Phi^i(x, u_i) \]  

(1)

In the above expression, the variables \( F^i \), \( x \) and \( u_i \) indicate respectively the N-dimensional output force generated by the controlled muscles, the state of the limb/environment interface and the controller's command variable\(^1\). We limit our discussion to the static impedance, that is to the relation between force and position at steady state. Our main results however apply to the dynamic impedance as well.

We also make the simplifying assumption that the dimensions of the limb's configuration space do not exceed the dimensions of the position variable, \( x \). In this case, the net force field, \( F(x, u_1, \ldots, u_K) \), generated

\[^1\text{In this paper, we adopt the convention of using superscripts to indicate vector and matrix objects and subscripts to indicate different vector and matrix components. For example, in this notation the component of the force generated by the n-th controller along the m-th direction is } F^m_n.\]