A VARIABLE K PLANETARY BOUNDARY-LAYER MODEL

(Research Note)

PRASANTA K. MISRA
Whiteshell Nuclear Research Establishment, Atomic Energy of Canada Limited, Pinawa, Man., Canada

(Received 22 June, 1976)

Abstract. The steady-state, homogeneous and barotropic equations of motion within the planetary boundary layer are solved with the assumption that the coefficient of eddy viscosity varies as $K(Z) = K_0(1-Z/h)^p$, where $h$ is the height of the boundary layer and $p$ is a parameter which depends on atmospheric stability. The solutions compare favourably with observed velocity profiles based on the Wangara data.

1. Introduction

The recent trend in the study of turbulent flows is to avoid using an eddy-viscosity coefficient. This is because such a concept does not hold during a free convection, or for the low-level jet streams observed in a stable, stratified, planetary boundary layer. However, some very useful insights into boundary-layer flows can be obtained by the use of a realistic eddy viscosity coefficient. The author has developed such a model.

Based on the numerical experiment of Deardorff (1972) and the field and wind-tunnel data of Clarke (1970) and Howroyd and Slawson (1975), a realistic form of the eddy viscosity coefficient within the planetary boundary layer is assumed. This is given by,

$$K(z) = K_0(1-z/h)^p, \quad 1 \leq p \leq 2,$$

where $K(z)$ is the eddy viscosity coefficient, $h$ the height of the boundary layer, $K_0$ a constant, and $p$ a parameter which depends on stability. The equations of motion within a barotropic, homogeneous and steady boundary layer, expressed in complex variables as

$$i f(V - V_\alpha) = d/dz(K(z) dV/dz)$$

are solved with the boundary conditions,

$$\lim_{z \to h} K(z) dV/dz = 0$$

and

$$K(z) dV/dz = u_* \exp (i\alpha_0), \quad z = 0,$$

where $V$ is the complex velocity, $V_\alpha$ the complex geostrophic velocity, $u_*$ the friction velocity, $\alpha_0$ cross-isobaric angle, and $i = \sqrt{-1}$. The lower boundary, $z = 0$, refers to the top of the surface layer.
Solutions to the above equations are obtained as:

\[ V = V_g - iu_{z0}^2/\sqrt{K_0} \exp \left( i(\alpha_0 - \pi/4) \right)(1 - z/h)^{(1-p)/2} I_n(r)/I_{n+1}(r_0), \]  

where \( I_n \) and \( I_{n+1} \) are the modified Bessel functions of order \( n \) and \( (n+1) \), respectively, \( n = (p-1)/(2-p) \), and \( r \) is given by,

\[ r = r_0(1 - z/h)^{(2-p)/2}, \]  

where

\[ r_0 = 2/(2-p)\sqrt{fh^2/K_0}. \]  

Equation (5) guarantees the condition,

\[ V \to V_g \quad \text{as} \quad z \to h \]

provided that

\[ u_{z0}^2/fh \ll V_g. \]  

This condition is normally met within the planetary boundary layer.

From the analyses of Tennekes (1970), it can easily be shown that the expression \( \sqrt{fh^2/K_0} = \sqrt{2} \Lambda \) tends towards zero with increasingly unstable stratification, and becomes very large in a stable stratification of the planetary boundary layer. In view of this, it is conjectured that the solution for an unstable stratification corresponds to

\[ r_0 \to 0 \quad \text{and} \quad p \to 1 \]  

and, for a stable stratification, it is

\[ r_0 \to \infty \quad \text{and} \quad p \to 2. \]  

Also,

\[ r_0 = 2n/(p - 1)\sqrt{fh^2/K_0}. \]  

Therefore, \( r_0 \gg n \) for a stable stratification.

The approximate solutions for unstable and stable stratifications are then obtained as:

\[ V = V_g - iu_{z0}^2/fh \exp \left( i(\alpha_0 - \pi) \right) \]  

and

\[ V = V_g - iu_{z0}^2/\sqrt{K_0} f \exp \left( i(\alpha_0 - 3\pi/4) \right) \exp \left( -(1+i)\Lambda z/h \right)/(1 - z/h)^{1/2}. \]  

In Equation (13), \( z \neq h \). However, this equation may be assumed to hold for a