Growth stress patterns in tree stems
A model assuming evolution with the tree age of maturation strains

M. Fournier, P. A. Bordonne, D. Guitard, Nancy, France,
T. Okuyama*, Nagoya, Japan

Summary. Growth stresses originate in maturation strains, induced in cambial layers during the differentiation and the maturation of new cells, impeded by the mass of the whole trunk. To predict stresses in a stem, one must add successive incremental stresses at successively created points of the growing trunk. Usual measurements of released strains at the surface of a stem do not give the evolution of maturation strains with growth. As the assumption that states that maturation strains are constant since the beginning of secondary growth leads to singularities near the pith, an empiric pattern of variation of maturation strains along the radius is proposed, using observations about relationships between released longitudinal strain and microfibril angle. Furthermore, assuming an elastic, orthotropic behavior law and a cylindrical, sufficiently long stem, residual stresses are computed and discussed.

For hardwoods, far enough from the pith, patterns of computed longitudinal and tangential stresses agree with distributions already stated by other authors although these stress components are limited near the pith. Computed radial stresses have lower levels than commonly admitted. On the other hand, stress-distributions in young softwoods are found very different, especially near the pith where the longitudinal component appears tensile and the tangential one compressive.

Introduction

Wood in standing trees undergoes internal stresses during the whole life of the tree. That means a potential ability of the material to strain or even to crack when processed (Archer 1986a; Kubler 1987).

Various methods allow the measurement of released longitudinal and tangential strains, $\varepsilon_L$ and $\varepsilon_T$, at the stem periphery. $\varepsilon_L$ is usually negative (shortening) and may exceed $10^{-3}$, $\varepsilon_T$ is positive, of the order of about $-2\varepsilon_L$. Accordingly, as $\varepsilon_T > -\nu_{LT}\varepsilon_L$ ($\nu_{LT}$: Poisson’s ratio), it can be assessed that in most standing trees, longitudinal component of stress is tensile and the tangential one is compressive (Ferrand 1981). However this assessment may not apply in case of reaction wood and for juvenile wood (i.e. small diameter logs).

* The authors wish to thank Prof. R. R. Archer, Department of Civil Engineering, University of Massachusetts at Amherst, USA, for his valuable suggestions and critical readings of the manuscript.
The living tree takes advantages of these stresses: tensile longitudinal stresses protect sapwood against excessive wind induced compressive bending stresses, and compressive tangential stresses counteract frost or drying cracks propagation (Kubler 1987; Archer 1986a).

To describe the origin of growth stresses, we will use the following assumption, which agrees with anatomical, physiological and chemical observations, according to many authors (e.g. Boyd 1972; Archer et al. 1974; Archer 1987; Bamber 1987): During radial growth, cells of outermost layers, just after differentiation, have a tendency to shrink in fiber direction and swell in the transverse one. These strains are impeded by the central part of the trunk, that leads to internal stresses in the whole tree.

**Cumulative stresses during radial growth**

It is assumed that every point of a layer is strongly stressed just after it is created (maturation strains are impeded). Per contra, an incremental distribution of stresses is added to previously set equilibrium in matured xylem so that the stem is always auto-equilibrated. So, two phenomena lead to stress in a tree stem: maturation strains in outermost layers, as well as radial growth.

Therefore, modelization of growth stresses in any point of a given trunk should be done in two steps:

- First, incremental strains and stresses due to the formation of a new layer should be evaluated when the geometry of the structure, the material behavior and the boundary conditions are known. A solution is computed (Appendix A), assuming simple hypothesis: axissymmetry of the problem; circular cylindricity of the stem; homogeneous, elastic and orthotropic behavior for wood.

- Then, incremental stresses at each material point due to a great number of successive layers can be superimposed. In the present paper, a method that deals with a finite number of layers has been applied as described in Appendix C.

In the vicinity of the axis of a tree trunk, the pith is not made of normal wood. In the incremental problem, orthotropic behavior leads to define a pith at least transversal isotropic (Maiti et al. 1968; Archer et al. 1974). Furthermore, all the previous works (Kubler 1959; Beck 1974; Archer et al. 1974; Gillis et al. 1979; Archer 1986 a) have shown that the axis is a singularity of the cumulative method itself, even in isotropic materials. Because these models superposed stresses due to successive infinitesimal layer, assuming, at each step, that stresses created in the outer layer are constant whatever the stage of growth (i.e. the external radius R). Thereby resulting stresses are functions of the reduced position r/R and patterns for stresses are similar at any time. Such an assumption may be correct if the new layer is created when the stem is large but if the central core has a small radius, and very low stiffness and strength as observed, the first layer cannot undergo high longitudinal tensile stresses for it could not be balanced by compressive stresses in the core. In this case, maturation strains are equivalent, as a mechanical phenomenon, to shrinkage or swelling of a wood pole (Tahani 1988).

The purpose here is to select appropriate parameters and propose a continuous evolution of maturation strains with the tree age, using relationships with the mi-