Anisotropy of the Reynolds stresses in a turbulent boundary layer on a rough wall

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Abstract The measured anisotropy invariants of the Reynolds stress tensor in a self-preserving rough wall turbulent boundary layer indicate that the anisotropy is significantly smaller than in a smooth wall layer.

1 Introduction

While the effect of surface roughness on the mean velocity distribution of a turbulent boundary layer is well documented, the effect on the turbulence structure is less well known. It is often assumed that, at sufficiently high Reynolds numbers, the turbulent motion outside the roughness sublayer (a region influenced by the length scales associated with the roughness elements and extending to about five times the roughness height), is independent of the wall roughness (Raupach et al. 1991). Although this hypothesis has received experimental support (see Raupach et al. for references), there is also evidence to the contrary. The measurements of Krogstad et al. (1992) and Acharya and Escudier (1987) show that the wall-normal turbulence intensity is significantly higher on a mesh roughness (k-type*) then a smooth wall. Antonia and Luxton (1971) reported higher streamwise and wall-normal turbulence intensities over a two-dimensional transverse bar roughness than a smooth wall. Increased magnitudes of all turbulence intensities have also been reported for a spherical roughness (Pimenta et al. 1979, Hosni et al. 1988). Krogstad and Antonia's (1994) two-point velocity correlations indicate differences in the large scale structure between rough and smooth walls. These differences, in turn, suggest possible differences in the anisotropy of the Reynolds stresses between the two flows. Apart from being important in the context of developing turbulence models, a knowledge of this anisotropy should be useful in determining the sensitivity of the turbulence structure to different boundary conditions.

2 Experimental details

Measurements were made in a zero-pressure gradient turbulent boundary layer on a rough wall. The roughness consisted of a 3.53 m long woven stainless steel screen of thickness k = 1.55 mm with a wire spacing/diameter ratio of 4.61. The boundary layer was tripped using a 4 mm diameter tripping rod followed by a 150 mm wide strip of 40 grit abrasive paper. A detailed description of the equipment and flow conditions are given by Krogstad et al. (1992). The present measurements were made using a single hot wire, as well as X- and V-probes. In all cases, 5 μm diameter Pt-10% Rh Wollaston wires, partially etched to a length of 1 mm, were used. For the X-probe, the separation between the two wires was about 1 mm. The included angles of the X- and V-probes were ~106° and 99° respectively. All hot wires were operated with in-house constant temperature anemometers at a overheat ratio of 1.5. Measurements were made at x = 3.23 m from the entrance of the working section and at a free stream velocity U∞ of 20.1 m/s. Under these conditions, the Reynolds numbers based on the momentum thickness, θ, and k were respectively Reθ (≡ Uθ/ν) and k = (kU2/ν) ≈ 104, with a boundary layer thickness θ ≈ 90 mm. The basic flow parameters were inferred from the mean velocity profiles using the optimization scheme described in Krogstad et al. (1992).

3 Results and discussion

In this note, we examine and quantify the Reynolds stress anisotropy using the anisotropy invariants introduced by Lumley and Newman (1977) and Lumley (1978). A measure of the anisotropy is provided by the Reynolds stress anisotropy tensor $b_{ij} = \frac{u_i u_j}{k} - \delta_{ij}$, where $u_j = u, v, w$ (u, v, and w are velocity fluctuations in the streamwise x, wall normal y, and spanwise z directions respectively), $k = \frac{u'^2}{2}$ is the average turbulent kinetic energy and $\delta_{ij}$, the Kronecker delta, = 1 when $i = j$ and = 0 when $i \neq j$. The anisotropy invariant map or AIM, which is a cross-plot of the second (II = $-b_{ij} b_{ji}/2$) versus third (III = $b_{ij} b_{jk} b_{kl}/5$) invariants of $b_{ij}$, provides a useful framework for examining the flow anisotropy.

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1 For a k-type roughness, the roughness function, $\Delta U/\langle U \rangle$, is determined by the roughness height k (Perry et al. 1969); here $U_\tau$ is the friction velocity.
with the corresponding components calculated using the data of Antonia et al. The data of EJ are used here since number effects should be small when generally good agreement with those of Antonia et al. (1990) and the DNS Reynolds number effects should be small when $R_e$ is greater than about 3000 (Antonia et al.). The data of EJ are used here since $w^+$ was not measured by Antonia et al.

Profiles of $b_{11}$, $b_{22}$, $b_{33}$ and $b_{13}$ are shown in Fig. 2, together with the corresponding components calculated using the data of EJ and Spalart. In the inner region, $b_{11}$ and $b_{22}$ are both significantly smaller over the rough than the smooth wall. In the outer region ($y^+ < 15$), $b_{11}$ and $b_{22}$ are closer to the isotropic value of zero than $b_{13}$, and $s$ refer to the present rough wall data and the EJ smooth wall data respectively. The ratio ($b_{11}-b_{13}$)/$b_{11}$ varies between 21% at $y^+ = 0.2$ to 33% at $y^+ = 0.6$. Over the same range, ($b_{22}-b_{13}$)/$b_{22}$ varies in the range 30% to 47%. This significant reduction in $b_{22}$ is consistent with the observed increase of $v^+$, due to the reduced damping of a rough wall (Krogstad et al. 1992). No marked difference can be observed between $b_{13}$ and $b_{13}$ in the outer region. It seems that the roughness reduces the anisotropy of $u^+$ and $v^+$, both in the inner and the outer region. This, in turn, implies the existence of a close interaction between the two regions.

A plot of $-II$ vs $III$ is shown in Fig. 3 (the data of EJ are not shown here as the first measurement location is at $y^+ = 0.05$). The maximum smooth wall values of $-II$ and $III$, which correspond to the maximum anisotropy, occur at $y^+ = 8$. The magnitudes of the rough wall invariants are quite close to the bottom cusp implying that the Reynolds stress tensor is close to isotropy. This behaviour reflects the smaller magnitudes of $b_{22}$ and $b_{13}$ over the rough wall.

Suzuki and Kasagi (1993) have calculated the diagonal components of $b_{ij}$ for a flow over a riblet surface. Compared to the present roughness, the drag augmenting riblet geometry $s^+ = 31$, $h^+ = 19$; $s$ and $h$ are riblet spacing and height respectively) represents only a minor modification to the surface. Yet, the maximum values of $-II = 0.143$ and $III = 0.0156$ are significantly smaller than on a smooth wall ($-II = 0.221$, $III = 0.0374$). This implies that $b_{ij}$ is a relatively sensitive indicator of the changes in the turbulence structure which result from a change in surface. Although Suzuki and Kasagi's flow was in the transitional regime, the trend of the invariants (not shown here) indicate an unambiguous tendency towards isotropy as a fully rough flow is approached.

In order to focus on the data near the bottom cusp, an enlargement of this region is shown in the inset (for clarity, a bold line has been used instead of symbols). Also shown are the invariants of the smooth wall data of EJ. The arrows point in the direction of increasing $y^+$. For the rough wall, the anisotropy is largest ($-II = 0.048$, $III = 0.0031$) at $y^+ = 0.05$, where $u^+$ is maximum. At smaller $y^+$, the trend of the data is towards the left (axisymmetric turbulence) boundary. This trend, which contrasts with the approach towards a two-component boundary for the smooth wall layer, should be verified with a more reliable measurement technique (e.g. LDA, PIV) because of the shortcomings of hot wires near a wall. At $y^+ = 0.2$, $-II = 0.034$