KNOWLEDGE AND NECESSITY*

C. I. Lewis's system S5 has a claim to being the propositional calculus of absolute necessity. But it is also clear that we lack a received and adequate justification of the laws for iterated modalities which are distinctive of both S4 and S5. We believe that granted the assumption of certain traditional doctrines about a priori knowledge, the characteristic axioms of S4 and S5 about iterated modalities can be derived. We shall present systems in which such derivations are carried out.

We begin with a language $L$. $L$ has an infinity of sentence letters, the truth functional connectives "~" and "→" and the necessity operator "□". $L$ also has two one-place sentence operators "K" and "A" which we read as "it is known that" and "it is known a priori that". The formation rules of $L$ are as usual. In addition to all the truth functionally valid wfs of $L$, our axioms are selected from these eight: For any wfs $p$ and $q$ of $L$,

(1) $□p → p$
(2) $□(p → q) → (□p → □q)$
(3) $Kp → p$
(4) $Ap → Kp$
(5) $p → □Kp$
(6) $□Ap → □p$
(7) $K□p → A□p$

Our rules of inference are modus ponens and necessitation. One of our systems, $A1$, includes 1–7; the other, $A2$, includes 1–8. These systems are defined as usual.

Some comments on the axioms are in order. Axioms 1 and 2 state basic properties of necessity, and axioms 3 and 4 are equally uncontroversial.
principles about knowledge. Axioms 5–8 embody part of a tradition about \textit{a priori} knowledge and the modalities.

The referee of the first draft of this note pointed out that in the system whose axioms are 1–3, 5 and "\(K(p \& q) \rightarrow (Kp \& Kq)\)" it is provable that

\((*)p \rightarrow Kp.\)

The crucial step is the instance "\((p \& -Kp) \rightarrow \Box K(p \& -Kp)\)" of Axiom 5. (For details see Fitch’s paper cited in the bibliography.) That \((*)\) is provable in this system is a revealing fact about the relations between knowledge and necessity; (note that all consideration of the \textit{a priori} can be omitted from this system). We think that the music should be faced. Axiom 5 is a apparently weak thesis of idealism or verificationism; a transcendental idealist like Kant might thus have accepted axiom 5. But in the presence of obvious truths, \((*)\) is deducible from 5. \((*)\) is obviously false and is an objectionably strong thesis of idealism (rather than an objectionably strong thesis of extensionalism). Therefore Axiom 5 is false; there are truths which absolutely cannot be known. This last is a very strong thesis of realism for which, on pain of paradox, there would not be a constructive proof. As thorough-going realists and anti-verificationists, we are very pleased by the strong thesis of realism. Axioms 7 and 8 state traditional views to the effect that knowledge of modality is always \textit{a priori}; Kant states essentially axiom 7 at B3 in \textit{The Critique of Pure Reason}, and 6 at B4. One of us (Hart) denies both 7 and 8.\footnote{These objections are due to Kripke. Sometimes one comes to know that \(\Box\ p\) by inferring it from one’s knowledge that \(p\) and that \(p \rightarrow \Box p\). Here the second is presumably \textit{a priori} knowledge, but the first may often be \textit{a posteriori} knowledge, and in such cases tradition has it that one knows the conclusion \textit{a posteriori}. If so, 8 is false. Similarly, in a Kripke-style essentialist argument (many of which we accept) one reasons from knowledge that \(p\) and that \(p \rightarrow \Box p\) to knowledge that \(\Box p\). Here, while the second premise is known \textit{a priori}, the first is known \textit{a posteriori}; and, again, in such cases tradition decrees that the conclusion is known \textit{a posteriori}. If so, 7 is false. Axiom 6 is difficult. Kripke’s well-known meter rod example purports to be a case in which \(Ap\) but \(\sim \Box p\), but we are unsure whether the example makes its point.}

We now show that \(A_1\) and \(A_2\) are extensions of \(S4\) and \(S5\) respectively. The first claim follows from the fact that \(\Box p \rightarrow \Box \Box p\) is a theorem of \(M\). To see this, suppose that \(\Box p\). By axiom 5, \(\Box K\Box p\). We can deduce from 7 by